Negative Polarity Items in definite superlatives*

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1 Introduction

The absolute/relative ambiguity of definite superlative noun phrases in English has been studied from a variety of theoretical perspectives, including seminal work by Ross (1964), Szabolcsi (1986) and Heim (1985, 1999). In this article, we contribute to this discussion by examining the distribution of Negative Polarity Items (NPIs) in and around superlative descriptions, especially on their relative readings. In particular, we attempt to account for the two facts below. The first, Generalization I, is not new (see, e.g., von Fintel 1999, Gajewski 2010, and Howard 2014), but to our knowledge, Generalization II has not been previously documented or analyzed. Under common assumptions about the licensing requirements of NPIs, we will show that both of these observations bear on established debates concerning the semantics and composition of phrases containing superlative adjectives.

Generalization I

NPIs are licensed *inside* English definite superlative noun phrases, whether the superlatives are interpreted absolutely, as in (1a) or relatively, as in (1b). This is in contrast with plain (non-superlative) singular definite descriptions like (2), where NPIs are generally ungrammatical.

Generalization II

NPIs are also licensed *outside* English definite superlative noun phrases, but only when those superlatives are interpreted relatively, as in (3). Again this contrasts with plain definite descriptions like (4), which never license NPIs elsewhere in an utterance.[1]

(1) Mary read the longest book that anyone read.

a. ✓ **Absolute reading**: Mary read the unique book that was longer than any other book that was read

b. ✓ **Relative reading**: Mary read a book that was longer than any book read by anyone other than Mary

(2) *Mary read the long book that anyone read.

(3) Mary ran the club that gave the largest prize to any freshman.

a. ✗ **Absolute reading**: Mary ran the club that gave some freshman the prize larger than any other prize

b. ✓ **Relative reading**: Mary ran the club that gave some freshman a prize larger than any prize given to any freshman by any other club

(4) *Mary ran the club that gave the large prize to any freshman.

Section 2 describes the problems posed by Generalization I for many well-known theories of superlatives, largely following the discussion in Howard 2014. Section 3 then describes new problems posed by Generalization II for the potential solution that Howard advances in light of Generalization I. Sections 5 and 6 attempt to address these problems. In Section 5, we offer a conservative reanalysis of Howard's data that preserves as much of his insight as possible, but ultimately fails to deliver appropriate denotations. So in Section 6, we propose an alternative theory of superlatives, similar to that in Bumford 2017a, and show that it fares better.

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1 Note that the descriptions in (3) and (4) have irrelevant Free Choice readings on which they describe a club that, as a rule, gave all freshmen the large(-est) prize. We will ignore such readings throughout the manuscript; all diacritics are intended as judgments when the relevant words are interpreted as NPIs.

2 Two classic superlative challenges

Superlative descriptions systematically license NPIs. Superlative descriptions are also systematically ambiguous. Both of these things are well-known and well-studied, though they are rarely studied at the same time. And it turns out that there is quite a bit of tension between popular accounts of the two phenomena. So we begin by discussing two common analyses, one developed to capture the distribution of NPIs, the other to derive the ambiguity. We show that they are incompatible.

2.1 Superlatives and NPIs

First, the NPI problem. Consider the contrast in (5). The subjects in both (5a) and (5b) are syntactically definite noun phrases (i.e., DPs whose surface determiner position is occupied by a definite determiner), but (5a) contains the superlative adjective *highest*, where (5b) does not. Correspondingly, the NPI *ever* is acceptable in the former and unacceptable in the latter. The subject in (5a) is called an absolute superlative: a certain grade is presupposed to be higher than all of the other grades, and the description refers to it.

(5) a. The highest grade John (ever) got was kind of low.
b. The grade John (*ever) got was kind of low.

We assume that (6b) is the meaning of the definite determiner, along the lines of Sharvy 1980, where w is a world of evaluation and g is an assignment function. The first argument, K, is the implicit, context-dependent domain restriction of the determiner. The second argument, P, is a polymorphic property, a function from things of type σ to truth values, where σ is any type. For perspicuity, we will often suppress the K argument, assuming the domain to be restricted to things that are relevant. The denotation in (6b) is designed to apply to singular, plural, and mass noun phrases alike, though in this paper we will only be concerned with singular descriptions. In this case, P will be a property of atomic entities none of which are parts of one another, and the presupposition will amount simply to uniqueness: $\exists! x \in K\,[P(x)]$.

(6) a. $\max(K, P) \coloneqq \{x \in K \mid P(x) \wedge \forall y \in K\,[P(y) \rightarrow y \sqsubseteq x]\}$
b. $[\![\text{the}]\!]^{w,g} \coloneqq \lambda K \lambda P\colon |\max(K, P)| = 1.\, \lambda Q.\, Q\,(\iota x\,[x \in \max(K, P)])$

Regarding NPI-licensing, we assume the theory developed in Ladusaw 1979, refined in von Fintel 1999, and further refined in Lahiri 1998, Guerzoni & Sharvit 2007, and Gajewski & Hsieh 2014, according to which NPIs are subject to the distributional constraint in (7c). The reader is referred to these authors for independent motivation and applications to other kinds of presuppositional licensers. Note that this constraint expresses a *necessary condition* for NPI grammaticality. There are presumably many additional requirements — e.g., that an NPI should be pronounced after its licenser — that play a role in judgments of acceptability, but we do not attempt to spell out a complete theory. The first point we wish to make is merely that many analyses of superlatives do not even clear this monotonicity hurdle.[2]

(7) a. **Polymorphic Strawson Entailment**: $X \overset{\text{ST}}{\Rightarrow} Y$ iff either (i) or (ii) is true.
(i) X and Y are truth values, and $X \rightarrow Y$
(ii) X and Y are functions of type (σ, τ), and $X(a) \overset{\text{ST}}{\Rightarrow} Y(a)$ for every a in the domain of both X and Y

2 We should also say that everything in this paper is intended to comment on the felicity of *weak* NPIs like *any* and *ever* in superlative descriptions. Quite mysteriously, superlatives are also known to license at least some *strong* NPIs like *in years*.

(i) a. Erin is the tallest girl John has seen in years [from Gajewski 2007]
b. The tallest girl John had seen until Friday walked in the room

We are not aware of any formal explanation for this, including in work expressly aimed at accounting for strong NPIs in and around presuppositional operators (e.g., Gajewski 2011), and we will unfortunately not have any progress to report.

b. A function f is **Strawson Downward Entailing** (SDE) iff $f(Q) \overset{ST}{\Rightarrow} f(P)$ whenever $f(P)$ and $f(Q)$ are both defined and $P \Rightarrow Q$. A function f is **Strawson Upward Entailing** iff $f(P) \overset{ST}{\Rightarrow} f(Q)$ whenever $f(P)$ and $f(Q)$ are both defined and $P \Rightarrow Q$.

c. An NPI is **licensed** only if it occurs in the scope of an operator α such that $[\![\alpha]\!]^{w,g}$ is SDE and not SUE, for any w and g. We sometimes refer to α as a licenser of the NPI.

The LF and meaning of (5b) are given in (8), as an example. The unacceptability of the NPI in the restrictor is explained by the fact that $[\![\text{the}]\!]^{w,g}$ is both SDE and SUE on its first (explicit) argument. This is guaranteed by the fact in (9), together with the compositionality of $[\![\cdot]\!]^{w,g}$. If there is exactly one linguist in K and exactly one phonologist in K, then they are the same person, and so any sentences that differ only in how they describe this individual will have the same denotation, and thus be mutually entailing.

(8) a. [the grade [2 [John got t_2]]] was kind of low

b. $[\![(8a)]\!]^{w,g}$ is defined only if $[\![\text{the}]\!]^{w,g}\left(\lambda x.\, \text{grade}_w(x) \wedge \text{got}_w(x)(\text{j})\right)$ is defined, i.e., only if John received exactly one grade.
If defined, $[\![(8a)]\!]^{w,g}$ is true iff $[\![\text{kind of low}]\!]^{w,g}\left(\iota x\, \left[\text{grade}_w(x) \wedge \text{got}_w(x)(\text{j})\right]\right)$.

(9) If $[\![\alpha]\!]^{w,g} \Rightarrow [\![\beta]\!]^{w,g}$ and both $[\![\text{the } \alpha]\!]^{w,g}$ and $[\![\text{the } \beta]\!]^{w,g}$ are defined, then $[\![\text{the } \alpha]\!]^{w,g} = [\![\text{the } \beta]\!]^{w,g}$.

Whether or not the NPI in (5a) is licensed thus depends on the denotation assigned to the superlative. For starters, let us compare the predictions of the superlative morpheme *est*$^{\text{vF}}$ in (10), based on von Fintel 1999 and Gajewski 2010, with the predictions of the superlative morpheme *est*$^{\text{H1}}$ in (11), based on Heim 1999. Both denotations take as argument a relation between degrees and individuals R in addition to an individual x, and both require this individual to stand in the R relation to a degree that no other individual in the domain of R does. In other words, they both require there to be a degree that sets x apart from all other elements in the domain of R.

(10) $[\![\text{est}^{\text{vF}}]\!]^{w,g} := \lambda R \lambda P \lambda x\colon P(x) \wedge \exists d[R(d)(x)]$
$\quad .\, \exists d[\{x\} = \{x' \mid P(x') \wedge R(d)(x')\}]$

(11) $[\![\text{est}^{\text{H1}}]\!]^{w,g} := \lambda R \lambda x\colon \exists d[R(d)(x)]$
$\quad .\, \exists d[\{x\} = \{x' \mid R(d)(x')\}]$

Where the analyses differ is in whether the nominal complement of the superlative is taken as an independent argument, P, or whether it is simply absorbed as part of the measuring relation R. This difference typically has no effect on the predicted meanings. For instance, (12a) provides the LF of (5a) according to the *est*$^{\text{vF}}$-proposal. The superlative morpheme applies first to its gradable complement *high*, of type $(d,(e,t))$, and then to the property-denoting nominal restrictor *grade John got*. (12b) provides the LF of (5a) according to the *est*$^{\text{H1}}$-proposal. The superlative morpheme takes scope over the rest of the $(d,(e,t))$-type expression formed by abstracting over its degree-denoting trace. These two LFs have the same denotation, shown in (14). Note that we assume that gradable adjectives are downward monotone (see, e.g., Cresswell 1976). Accordingly, *high* denotes the relation that holds between any individual x and degree d if x's height is at least d, as in (13).

(12) a. [the [high *est*$^{\text{vF}}$] [grade [2 [John (ever) got t_2]]]] was kind of low

b. [the [*est*$^{\text{H1}}$ [1 [high-t_1 [grade [2 [John (ever) got t_2]]]]]]] was kind of low

(13) $[\![\text{high}]\!]^{w,g} := \lambda d \lambda x.\, \text{ht}_w(x) \geq d$

(14) $[\![(12a/b)]\!]^{w,g}$ is defined only if $\exists! x \exists d\left[\{x\} = \left\{x' \;\middle|\; \text{grade}_w(x') \wedge \text{ht}_w(x') \geq (d) \wedge \text{got}_w(x')(\text{j})\right\}\right]$, i.e., only if one of John's grades is higher than all the others.
If defined, $[\![(12a/b)]\!]^{w,g}$ is true iff $[\![\text{low}]\!]^{w,g}\left(\iota x\, \left[\exists d\left[\{x\} = \left\{x' \;\middle|\; \text{grade}_w(x') \wedge \text{ht}_w(x') \geq d \wedge \text{got}_w(x')(\text{j})\right\}\right]\right]\right)$.

But crucially these two proposed superlative denotations have different montonicity profiles. For any relation R, $[\![\text{est}^{\text{vF}}]\!]^{w,g}(R)$ is SDE (but not SUE). To see this, let $P \Rightarrow Q$, and let x be such that $[\![\text{est}^{\text{vF}}]\!]^{w,g}(R)(P)(x)$ and $[\![\text{est}^{\text{vF}}]\!]^{w,g}(R)(Q)(x)$ are both defined. Then we have the following simple proof.

1. Assume $[\![\text{est}^{\text{vF}}]\!]^{w,g}(R)(Q)(x)$ is true.
2. By definition of $[\![\text{est}^{\text{vF}}]\!]$, then, $\exists d\,[\{x\} = \{x' \mid Q(x') \wedge R(d)(x')\}]$. That is, for some d^*:
   (i) $\{x\} \subseteq \{x' \mid Q(x') \wedge R(d^*)(x')\}$, and
   (ii) $\{x\} \supseteq \{x' \mid Q(x') \wedge R(d^*)(x')\}$
3. From the first conjunct, it follows that $R(d^*)(x)$. And since $P(x)$ holds by assumption, $\{x\} \subseteq \{x' \mid P(x') \wedge R(d^*)(x')\}$.
4. From the second conjunct, it follows that $\{x\} \supseteq \{x' \mid Q(x') \wedge R(d^*)(x')\} \supseteq \{x' \mid P(x') \wedge R(d^*)(x')\}$, given the assumption that $P \Rightarrow Q$ (i.e., $\{z \mid Q(z)\} \supseteq \{z \mid P(z)\}$), and the fact that $A \cap Z \supseteq B \cap Z$ whenever $A \supseteq B$.
5. So $\exists d\,[\{x\} = \{x' \mid P(x') \wedge R(d)(x')\}]$, namely d^*
6. By definition of $[\![\text{est}^{\text{vF}}]\!]$, then, $[\![\text{est}^{\text{vF}}]\!](R)(P)(x)$ is true.
7. Thus, $[\![\text{est}^{\text{vF}}]\!](R)(Q) \overset{\text{ST}}{\Rightarrow} [\![\text{est}^{\text{vF}}]\!](R)(P)$, completing the proof.

To see that $[\![\text{est}^{\text{vF}}]\!]^{w,g}([\![\text{high}]\!]^{w,g})$ is not SUE, simply set $P := \lambda y.\, y \in \{1,2\}$ and $Q := \lambda y.\, y \in \{1,2,3\}$. Clearly $P \Rightarrow Q$, but $[\![\text{est}^{\text{vF}}]\!]^{w,g}([\![\text{high}]\!]^{w,g})(P)(2)$ is true while $[\![\text{est}^{\text{vF}}]\!]^{w,g}([\![\text{high}]\!]^{w,g})(Q)(2)$ is false.

Since the NPI in (14a) occurs in the scope of *highest*, the est^{vF} denotation can claim to predict its grammaticality. By contrast, the est^{H1} denotation does not predict this, because $[\![\text{est}^{\text{H1}}]\!]^{w,g}$ is not SDE. This is shown by the model below, with three books A, B, and C, and two relations R and S.

$R := \lambda d \lambda x.\, \text{length}(x) \leq d$
$S := \lambda d \lambda x.\, R(d)(x) \wedge d \leq 300$

x	length(x)	$\{d \mid R(d)(x)\}$	$\{d \mid S(d)(x)\}$
A	500	$\{d \mid d \leq 500\}$	$\{d \mid d \leq 300\}$
B	300	$\{d \mid d \leq 300\}$	$\{d \mid d \leq 300\}$
C	100	$\{d \mid d \leq 100\}$	$\{d \mid d \leq 100\}$

Clearly $S \Rightarrow R$. And $[\![\text{est}^{\text{H1}}]\!]^{w,g}(R)(\text{A})$ is true, since, for instance, 500 is a length that sets A apart in the R column from every other book. That is $\{\text{A}\} = \{x' \mid R(500)(x')\}$. But $[\![\text{est}^{\text{H1}}]\!]^{w,g}(S)(\text{A})$ is false, since there's no degree that only A stands in the S relation to; A and B have the same degrees in the S column. So $S \Rightarrow R$ does not guarantee that $[\![\text{est}^{\text{H1}}]\!]^{w,g}(R) \overset{\text{ST}}{\Rightarrow} [\![\text{est}^{\text{H1}}]\!]^{w,g}(S)$.

2.2 Superlatives and scope ambiguity

On the other hand, est^{H1} leads quite naturally to an account of the perceived ambiguity in sentences like (15).

(15) John got the highest grade
   a. ✓ **Absolute**: John got a perfect score (the grade higher than any other grade)
   b. ✓ **Relative**: John got a higher grade than anyone else did

Both readings are predicted if est^{H1} is able to take scope outside of its containing noun phrase. In the relative LF, movement of est^{H1} across *the* is hypothesized to neutralize the usual uniqueness presupposition of the

definite determiner; on the relative reading, it need not be the case that one of the received grades was in fact higher than all of the others. This is an admittedly awkward detail, but since it is the subject of much debate (see Szabolcsi 1986, Heim 1999, Sharvit & Stateva 2002, Coppock & Beaver 2014, Bumford 2017a, among others), we stipulate it here and press forward.

(16) $[\![\textit{the}]\!]^{w,g} := [\![\text{a}]\!]^{w,g}$

(17) a. **Absolute LF**: [John got [the [est^{H1}] [1 [high-t_1 grade]]]]
b. If defined, $[\![(17\text{a})]\!]^{w,g}$ is true iff $\text{got}_w(\iota x\,[\exists d\,[\{x\} = \{x' \mid \text{grade}_w(x') \wedge \text{ht}_w(x') \geq d\}]])(\text{j})$.

(18) a. **Relative LF**: [John [est^{H1} [1 [got *the* high-t_1 grade]]]]
b. If defined, $[\![(18\text{a})]\!]^{w,g}$ is true iff $\exists d\,[\{\text{j}\} = \{z \mid \exists y\,[\text{grade}_w(y) \wedge \text{got}_w(y)(z) \wedge \text{ht}_w(y) \geq d]\}]$

In a similar fashion, est^{H1} accounts for the unambiguously relative readings of adverbial superlatives, as in *Mary sang the loudest* — meaning Mary sang louder than anyone else — and of amount superlatives, as in *Mary read the most books* — meaning Mary read more books than anyone else.

(19) If defined, $[\![[\text{Mary } [est^{\text{H1}} [1 [\text{sang } \textit{the } \text{loud-}t_1]]]]]\!]^{w,g}$ is true iff
$\exists d\,[\{\text{m}\} = \{x' \mid x' \text{ sang } d\text{-loud in } w\}]$

(20) If defined, $[\![[\text{Mary } [est^{\text{H1}} [1 [\text{read } \textit{the } \text{many-}t_1 \text{ books}]]]]]\!]^{w,g}$ is true iff
$\exists d\,[\{\text{m}\} = \{x' \mid x' \text{ read } d\text{-many books in } w\}]$

It is hard to see how est^{vF} could account for such readings. The only real difference between est^{vF} and est^{H1} is the initial property-type argument that the former takes. As we saw, this argument was crucial to predicting the grammaticality of NPIs in absolute superlative descriptions, since it provided a genuinely negative environment, quarantined from the non-monotonic influence of the gradable predicate. But it is precisely this argument that seems to gum up the scope-taking works. Once est^{vF} has combined with its adjectival and nominal complements, it no longer denotes a higher-order function of the sort that is capable of quantifying over its own continuation, so it does not have the right type to raise. And yet est^{vF} cannot take scope before combining with these complements, as in the est^{H1} derivations above, because there is nowhere for it to land that provides the crucial (SDE) property-type argument P.

In any case, as far as we are aware, all of the literature since Szabolcsi 1986 that treats the absolute/relative distinction as a genuine scope ambiguity has abandoned the simple est^{vF} denotation, so for now, let us concede that it does not generate relative readings. In Sections 5 and 6, we return to the possibility that this is mistaken, and est^{vF}, or something similar, can account for relativity after all (see also Appendix B).

Now, if relative truth conditions are altogether beyond the reach of est^{vF}, then it seems fair to say, *a fortiori*, that est^{vF} does not explain how NPIs can be licensed in relative readings, as in (21).

(21) a. In this class, John got the highest grade anyone got.
b. Mary sang the loudest anyone sang.
c. Mary read the most books anyone read.

Likewise, even though est^{H1} *does* generate relative truth conditions, it does not predict the felicity of the NPIs in (21) any more than those in absolute descriptions (see Howard 2014 for more on this point). The reason is the same: est^{H1} only has one (functional) argument, R, and so if these NPIs are in the scope of the superlative, then they must occur as part of the constituent that delivers this R relation. But $[\![\text{est}^{\text{H1}}]\!]^{w,g}$ is not SDE with respect to this argument.

To sum up, $[\![\text{est}^{\text{H1}}]\!]^{w,g}$ generates both absolute and relative readings, but it is not SDE, and so does not manage to license NPIs in either case. In contrast, $[\![\text{est}^{\text{vF}}]\!]^{w,g}(R)$ is SDE for any R, but it derives only absolute

readings, and so it both undergenerates truth conditions and fails to explain the broad distribution of NPIs in the context of superlatives.

3 Superlatives and focus: A sentence-adverbial analysis

In this section we discuss a third potential denotation for the superlative morpheme, also due to Heim (1999). Like *est*$^{\text{H1}}$, this third analysis straightforwardly derives relative readings. Additionally, Howard (2014) has shown that it can be leveraged to provide an explanation for the felicity of NPIs in certain relative configurations, including the examples in (21). Unfortunately, as Howard notes, it is very hard to see how this sort of explanation could be extended to cover the basic absolute cases handled so smoothly by *est*$^{\text{vF}}$. We try our best to find suitable LFs, but do not succeed. Finally, we point out that even if the *est*$^{\text{H2}}$ morpheme that we introduce could ultimately provide a satisfactory account of absolute superlative descriptions containing NPIs, it would still fail to license NPIs that fall under Generalization II. That is, it will not explain the grammaticality of NPIs that occur outside the superlative noun phrase itself.

3.1 The adverbial superlative and monotonicity

Of the various alternative accounts of relative readings available in the literature, the following focus-based analysis explored in Heim 1999 has probably been the most influential. One of the motivations for this analysis is the fact that relative interpretations seem to vary with prosodic stress, as illustrated in (22) and (23).

(22) JOHN gave Mary the highest grade

a. ✓John gave Mary a higher grade than anyone else gave Mary

b. ✗John gave Mary a higher grade than he gave anyone else

(23) John gave MARY the highest grade

a. ✗John gave Mary a higher grade than anyone else gave Mary

b. ✓John gave Mary a higher grade than he gave anyone else

The superlative morpheme *est*$^{\text{H2}}$ is assumed to be a focus-sensitive sentence-level adverb, in the mold of adverbial *only*. The prejacent of *est*$^{\text{H2}}$ contains a focused expression, often called the *correlate*, and denotes a property of degrees P of type $(s,(d,t))$. The restrictor of the superlative C is the set of focus alternatives to its prejacent. Focus alternatives are computed in the style of Rooth 1992. In particular, $[\![\alpha \sim \text{C}]\!]^{w,g}$ imposes the requirement that C is a subset of the focus semantic value of α.[3] For any given degree property P and set of alternative degree properties C, $[\![\text{est}^{\text{H2}}]\!]^{w,g}(C)(P)$ is true iff there is a degree that sets the prejacent P apart from all of the other degree properties in C.

(24) $[\![\text{est}^{\text{H2}}]\!]^{w,g} := \lambda C \lambda P\colon P \in C \wedge \forall Q \in C\,[\exists d\,[Q(w)(d)]]$
$\quad .\, \exists d\,[\{P\} = \{Q \in C \mid Q(w)(d)\}]$

As before, movement of *est*$^{\text{H2}}$ to the sentence-adverbial position is assumed to reduce *the* to a determiner with merely existential force. For instance, the LFs in (25) and (26) deliver the relevant truth conditions of (22) and (23).

3 We use the conventions that $C \equiv [\![\text{C}]\!]^{w,g}$, whenever C is a free variable in an LF, and that $[\![^\wedge\alpha]\!]^{w,g} \equiv \lambda w'\colon [\![\alpha]\!]^{w',g}$ is defined. $[\![\alpha]\!]^{w',g}$.

(25) est^{H2}-C [[^ [1 [John$_{\mathbf{F}}$ gave Mary *the* high-t_1 grade]]] ~ C]

a. $[\![(25a)]\!]^{w,g}$ is defined only when $C \subseteq \{\lambda w'\lambda d.\, z \text{ gave Mary a } d\text{-high grade in } w' \mid z \in \mathcal{D}_e\}$, and $C \supseteq \{\lambda w'\lambda d.\, \text{John gave Mary a } d\text{-high grade in } w'\}$
If defined, $[\![(25a)]\!]^{w,g}$ is true iff
$\exists d\,[\{\lambda w'\lambda d.\, \text{John gave Mary a } d\text{-high grade in } w'\} = \{Q \in C \mid Q(w)(d)\}]$

(26) est^{H2}-C [[^ [1 [John gave Mary$_{\mathbf{F}}$ *the* high-t_1 grade]]] ~ C]

a. $[\![(26a)]\!]^{w,g}$ is defined only when $C \subseteq \{\lambda w'\lambda d.\, \text{John gave } z \text{ a } d\text{-high grade in } w' \mid z \in \mathcal{D}_e\}$, and $C \supseteq \{\lambda w'\lambda d.\, \text{John gave Mary a } d\text{-high grade in } w'\}$
If defined, $[\![(26a)]\!]^{w,g}$ is true iff
$\exists d\,[\{\lambda w'\lambda d.\, \text{John gave Mary a } d\text{-high grade in } w'\} = \{Q \in C \mid Q(w)(d)\}]$

So est^{H2} generates relative readings. What does it predict about NPIs? The derivations above would seem to suggest that an NPI like that in (21a) is contained in the superlative's prejacent P. After all, as Heim 1999 would have it (following Rooth 1992), the restrictor C is an implicit comparison class representing the formal question that the prejacent is taken to address, i.e., its focus alternatives. Unfortunately, for a fixed comparison class C, $[\![\text{est}^{\mathrm{H2}}]\!]^{w,g}(C)$ is not SDE (Howard 2014). To see this, consider the world w characterized by the model below. There are five singers and two degree properties per singer, P_z and Q_z. These are the ingredients that would be relevant for the sentences in (27).

(27) a. Cal sang the loudest yesterday
b. Cal sang the loudest this week

$P_z := \lambda w'\lambda d.\, z$ sang d-loud some time yesterday in w'
$Q_z := \lambda w'\lambda d.\, z$ sang d-loud some time this week in w'

z	$\{d \mid P_z(w)(d)\}$	$\{d \mid Q_z(w)(d)\}$
ann	$\{d \mid d \leq 20\}$	$\{d \mid d \leq 25\}$
bud	$\{d \mid d \leq 20\}$	$\{d \mid d \leq 30\}$
cal	$\{d \mid d \leq 35\}$	$\{d \mid d \leq 60\}$
deb	$\{d \mid d \leq 50\}$	$\{d \mid d \leq 55\}$
eve	$\{d \mid d \leq 30\}$	$\{d \mid d \leq 35\}$

In this world, yesterday's performance was nobody's loudest; in every row, there is a higher value in the Q column than in the P column. Now, let C contain P_z and Q_z for each singer z. That is, let it contain for each z the property corresponding to songs z performed some time yesterday, as well as the property corresponding to songs z performed some time this week:

$$C := \bigcup \Big\{ \{P_z, Q_z\} \;\Big|\; z \in \{\text{ann}, \ldots, \text{eve}\} \Big\}$$

Let $P \equiv P_{\text{cal}}$ and $Q \equiv Q_{\text{cal}}$. Clearly P and Q are both in C, so that $[\![\text{est}^{\mathrm{H2}}]\!]^{w,g}(C)(P)$ and $[\![\text{est}^{\mathrm{H2}}]\!]^{w,g}(C)(Q)$ are both well-defined. And clearly $P \Rightarrow Q$, since in any given world w', the decibels that Cal's singing reached yesterday are a subset of the decibels that Cal's singing reached at some point this week. Moreover, since there is a degree d^* (namely, 60) such that $Q(w)(d^*)$ is true while $Q'(w)(d^*)$ is false for any other $Q' \in C$, it follows that $[\![\text{est}^{\mathrm{H2}}]\!]^{w,g}(C)(Q)$ is true. But $[\![\text{est}^{\mathrm{H2}}]\!]^{w,g}(C)(P)$ is false; there is no degree in $P(w)$ that is not in $Q'(w)$ for any other $Q' \in C$. So $[\![\text{est}^{\mathrm{H2}}]\!]^{w,g}(C)(Q) \overset{\text{ST}}{\not\Rightarrow} [\![\text{est}^{\mathrm{H2}}]\!]^{w,g}(C)(P)$, and thus $[\![\text{est}^{\mathrm{H2}}]\!]^{w,g}(C)$ is not SDE.[4]

In other words, Cal might well be responsible for the loudest of all the songs performed the week, but that doesn't mean that Cal is responsible for the loudest of all the songs performed *yesterday*, even if Cal (and everybody else) did in fact sing at some point yesterday.

However, Howard (2014) points out that $[\![\text{est}^{\mathrm{H2}}]\!]^{w,g}$ *is itself* SDE. That is, it is SDE "on its first argument". The proof is exactly the same as the proof for $[\![\text{est}^{\text{vF}}]\!]^{w,g}$, except that the variables have higher types. Let P be an arbitrary degree property, type $(s,(d,t))$, and let C and D be sets of such degree properties such that $C \subseteq D$, with $P \in C$ and $P \in D$.

4 In fact, if P and Q are both in C, then $[\![\text{est}^{\mathrm{H2}}]\!]^{w,g}(C)(Q)$ actually entails that $[\![\text{est}^{\mathrm{H2}}]\!]^{w,g}(C)(P)$ is false, whenever $P \neq Q$!

1. Assume $[\![\text{est}^{\text{H2}}]\!]^{w,g}(D)(P)$ is true.
2. By definition of $[\![\text{est}^{\text{H2}}]\!]^{w,g}$, then, $\exists d\,[\{P\} = \{Q \in D \mid Q(w)(d)\}]$. That is, for some d^*:
   (i) $\{P\} \subseteq \{Q \in D \mid Q(w)(d^*)\}$, and
   (ii) $\{P\} \supseteq \{Q \in D \mid Q(w)(d^*)\}$
3. From the first conjunct, it follows that $P(w)(d^*)$.
4. From the second conjunct, it follows that $\{P\} \supseteq \{Q \in D \mid Q(w)(d^*)\} \supseteq \{Q \in C \mid Q(w)(d^*)\}$, given the assumption that $D \supseteq C$.
5. So $\exists d\,[\{P\} = \{Q \in C \mid Q(w)(d)\}]$, namely d^*.
6. By definition of $[\![\text{est}^{\text{H2}}]\!]^{w,g}$, then, $[\![\text{est}^{\text{H2}}]\!]^{w,g}(C)(P)$ is true.
7. Thus, $[\![\text{est}^{\text{H2}}]\!]^{w,g}(D)(P) \overset{\text{ST}}{\Rightarrow} [\![\text{est}^{\text{H2}}]\!]^{w,g}(C)(P)$, completing the proof.

Recognizing this, Howard hypothesizes that the grammatical NPIs in (21) must somehow be contained in a constituent that denotes this restrictor argument of the superlative. That is, where the traditional Heimian analysis of relative readings treats C as an implicit parameter generally associated with the focus structure of the superlative's prejacent, Howard suggests that in some cases this comparison class argument is actually expressed in the sentence. And in exactly these cases, NPIs might appear felicitously inside the constituent that expresses this comparison class. We now present a slightly amended version of that proposal, which we hope is faithful in all crucial respects. As stated in the introduction to this section, the goal in Section 4.1 will be to then work backwards from this analysis to try and use *est*$^{\text{H2}}$ to account for NPIs in *absolute* readings. But we will not succeed in this. And what's more, in Section 4.2 we will show that NPIs also appear felicitously inside constituents that cannot possibly be part of this comparison class argument.

4 Superlative ACD: Howard 2014

Suppose that in addition to the usual association-with-focus mechanism depicted above, there is also a syntactic means of forming a restrictor of the appropriate type for *est*$^{\text{H2}}$. As we've seen, the Roothian route never licenses NPIs, since its only SDE environment is an unpronounced parameter. But the second route ought to license them in the constituent that forms this restrictor. Howard (2014) proposes that this syntactic mechanism involves a kind of relative clause ellipsis familiar from comparative ACD, together with an Alternative Semantics approach to indefinites (cf. Kratzer & Shimoyama 2002), or at least to NPIs.

To formalize this, we first assume that sentences like those in (21) — based on Howard's crucial relative readings with NPIs — contain elliptical material in their relative clauses, as is common for analogous sentences with comparatives. For instance, where (28a) is often derived from an LF like that in (28b) (see, e.g., von Stechow 1984, Rullmann 1995), we might assume (29a) is derived from an LF along the lines of (29b).

(28) a. John sang louder than Mary sang
   b. [[er [1 [Mary sang ~~t_1-loud~~]]] [1 [John sang t_1-loud]]]

(29) a. Mary sang the loudest that anyone sang
   b. [[est [1 [anyone sang ~~t_1-loud~~]]] [1 [Mary sang t_1-loud]]]

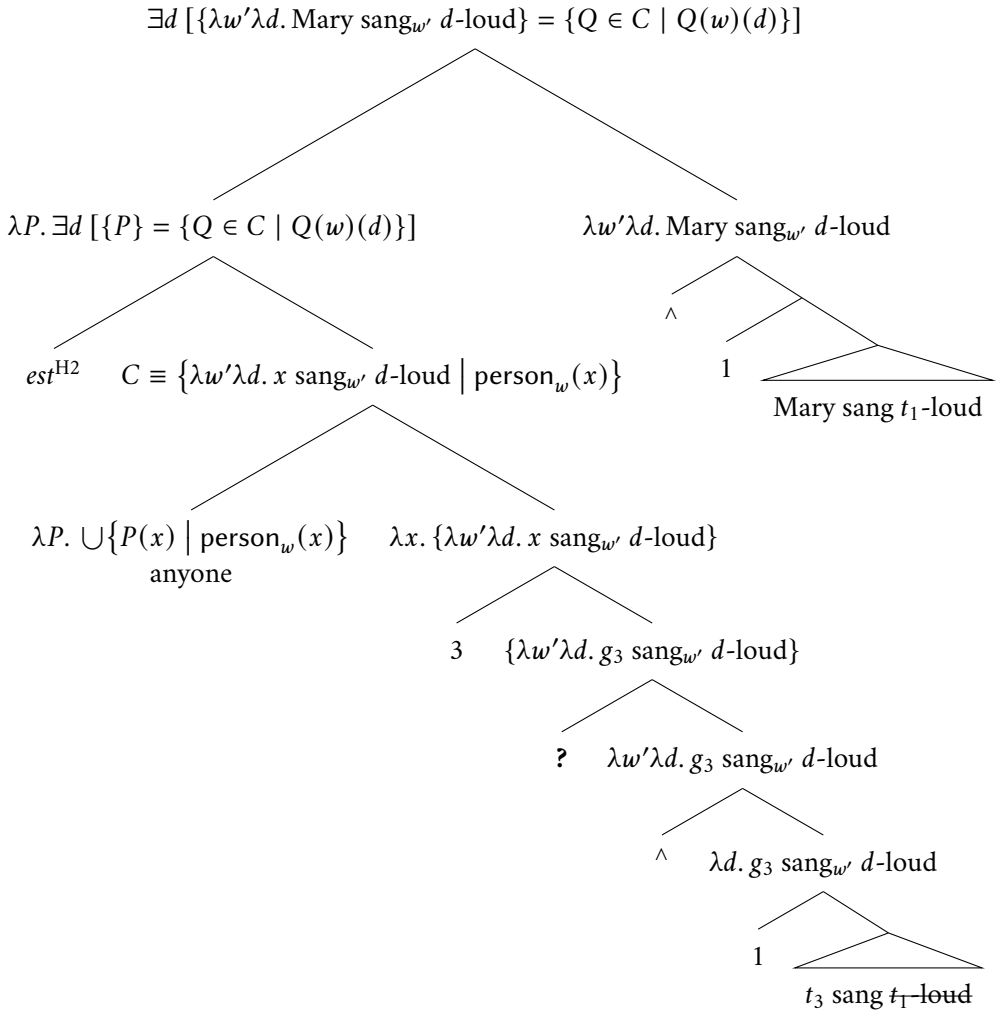
The difference between the comparative case of ACD and the superlative case is that the superlative's relative clause ought to denote a *set* of alternative degree properties (all those that the prejacent is taken to exceed), where the comparative's relative clause denotes a *single* competitor to its prejacent. So we follow Howard in borrowing Karttunen's (1977) singleton set-forming operator **?** (cf. also Partee's (1986) type-shifter IDENT), and in assuming that indefinite NPIs generate alternatives (cf. Krifka 1995, Chierchia 2013).

(30) a. $[\![\textbf{?}]\!]^{w,g} := \lambda Q.\, \{Q\}$

b. $[\![\text{anyone}]\!]^{w,g} := \lambda k.\, \bigcup \{k(x) \mid \text{person}_w(x)\}$

With this, we can derive the truth conditions of (29) as in (31). Because the NPI *anyone* is contained within the superlative's restrictor, it is correctly predicted to be grammatical. Analogous derivations involving *ever* are provided in Appendix A, which we set aside to avoid complications due to tense and aspect.

(31) $[\![\text{Mary sang the loudest that anyone sang}]\!]^{w,g}$



As Howard notes, ACD *per se* is not required for any semantic reason, but the presuppositions of $[\![\text{est}^{\text{H2}}]\!]^{w,g}$ do impose a very strong requirement on the relationship between the meanings of the restrictor and nuclear scope here. Recall that for $[\![\text{est}^{\text{H2}}]\!]^{w,g}(C)(P)$ to be well-defined, P must be an element of C. That is, the prejacent must actually be one of the degree properties up for comparison. When C is derived from the focus structure of the prejacent itself, this is guaranteed. But when C is generated compositionally from additional elements in the sentence, it is possible for this presupposition to fail. For instance, Howard observes that the sentences in (32) are deviant. The first would require that the degree property ($\lambda d \lambda w'$. Mary $\text{read}_{w'}$ d-many books) be among the set of degree properties $\{\lambda d \lambda w'.\, z$ $\text{published}_{w'}$ d-many books $\mid \text{person}_w(z)\}$, which is impossible; the second would require that ($\lambda d \lambda w'$. Mary $\text{sang}_{w'}$ d-loud) be in the set $\{\lambda d \lambda w'.\, z$ $\text{sang}_{w'}$ d-loud $\mid \text{baritone}_w(z)\}$, which is possible, but requires Mary to be a baritone.

(32) a. *Mary read the most books that anyone published

b. #Mary sang the loudest that any baritone sang

We will come back to this "matching" generalization in Section 7. First, we discuss some undergeneration issues of est^{H2} + ACD approach.

4.1 Superlative ACD and absolute readings

The denotation for est^{H2} was expressly designed to account for relative readings. But what, then, of *absolute* readings? It is certainly conceivable that relative superlatives are composed from est^{H2} while absolute superlatives are composed from est^{vF} (which is SDE on its (e,t)-argument, as we saw in Section 2.1). However, in the absence of independent evidence for the existence of two distinct underlying superlative morphemes, we would expect there to be a way to form absolute superlatives with est^{H2}.[5] The LF in (33) shows that this is technically possible if est^{H2} associates with a focused trace bound by the definite determiner (see also Romero 2013).

(33) John climbed the highest mountain

a. John climbed [the [3 [est^{H2}-C [[^ [1 [[t_3]$_F$ high-t_1 mountain]]] ~ C]]]]

b. $[\![$(33a)$]\!]$ is defined only when $C \subseteq \{\lambda w'\lambda d.\, z \text{ is a } d\text{-high mountain in } w' \mid z \in \mathcal{D}_e\}$
$C \supseteq \{\lambda w'\lambda d.\, g_3 \text{ is a } d\text{-high mountain in } w'\}$

If defined, $[\![$(33a)$]\!]$ is true iff

John climbed $\iota z \left[\exists d \left[\begin{array}{l}\{\lambda w'\lambda d.\, z \text{ is a } d\text{-high mountain in } w'\} \\ = \{Q \in C \mid Q(w)(d)\}\end{array}\right]\right]$

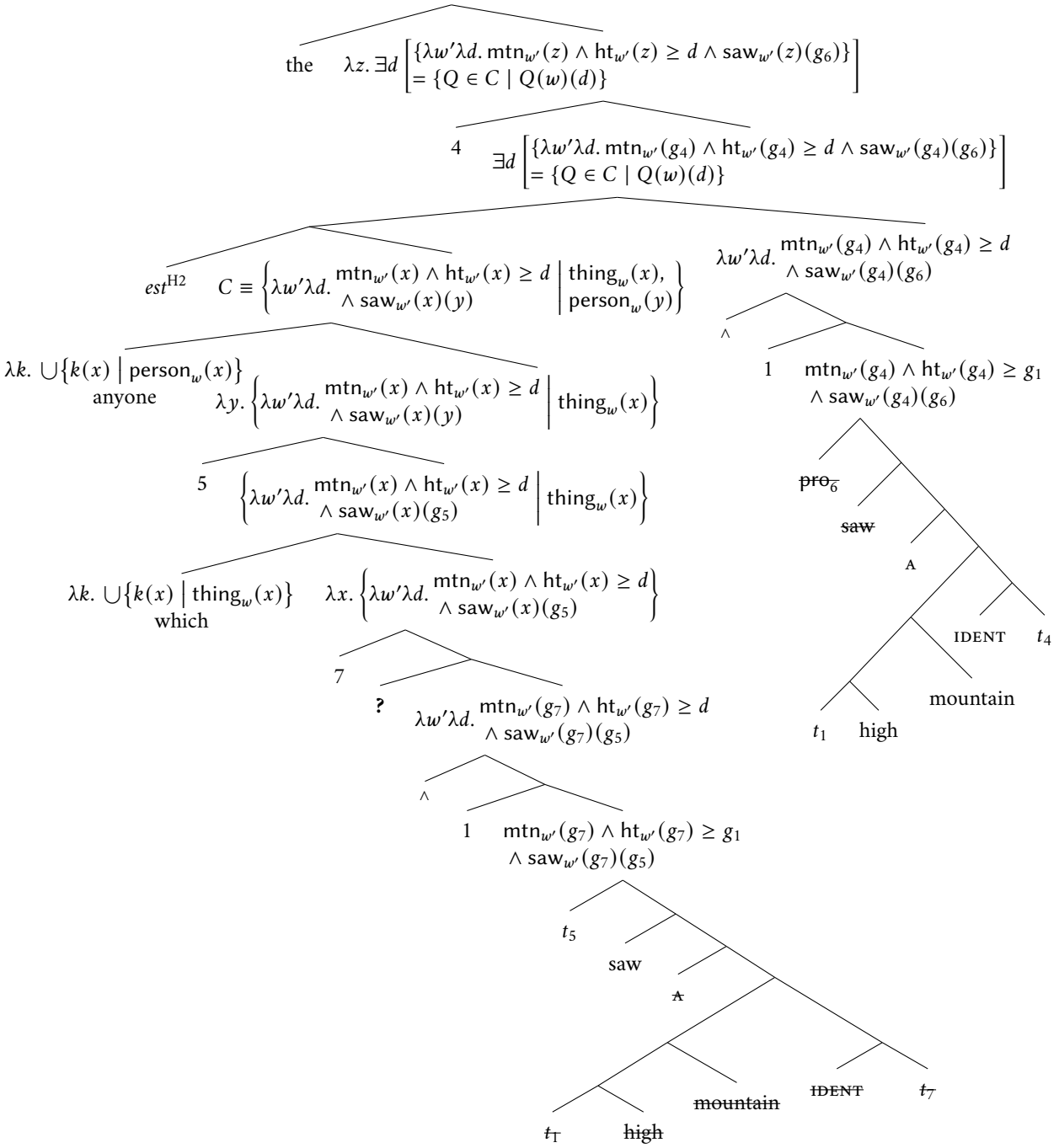
The truth conditions in (33b) express the intuitive meaning of (33), but any NPI in the nominal description of an LF like this would appear in the (non-SDE) scope of $[\![\text{est}^{H2}]\!]^{w,g}(C)$, rather than in the (SDE) restrictor C. Howard attempts to derive the absolute reading from other possible est^{H2}-based LFs that would place an NPI in the restrictor argument, but he ultimately abandons the effort.

We offer one additional unsuccessful stab at this, if only to demonstrate how elusive the proper LF would have to be. As we have seen, according to Howard, there are two ways to obtain relative readings when the superlative takes scope *outside* a morphologically definite noun phrase: association with focus, as in (22), and alternative generation via ACD, as in (31). Suppose the same two grammatical mechanisms are available when the superlative takes scope *inside* a morphologically definite noun phrase. That is, the restriction might be derived by association with focus, as in (33), or it might be derived via ACD in the presence of alternative-generators, as in (34). Note that we assume the elided pronoun pro_6, which is free in the prejacent, is bound by a widest-scope existential-closure operator. And we appeal to common A and IDENT type-shifters to facilitate composition with relative clause copies, in the spirit of the usual Trace Conversion techniques in, e.g., Fox 2003, Heim 2018. As elsewhere in this paper, struck out phrases are elided.

The sentence is predicted to be true iff, for some choice of g_6, John climbed the unique z such that (i) z is a mountain seen by g_6, (ii) z is higher than any other mountain seen by g_6, and (iii) z is higher than any mountain seen by anyone other than g_6. If there is such a g_6, there is only one; only one person can have seen a mountain higher than any mountain seen by anyone else. And if there is such a g_6, then the absolute superlative description picks out the highest mountain that g_6 saw (so long as there is, indeed, a single mountain higher than any others among this person's viewing history). What's more, the NPI *anyone* is now in the restrictor of est^{H2}, and so is correctly predicted to be grammatical.

5 We are aware of languages that have more than one superlative morpheme, but we are unaware of languages where the two morphemes are used to distinguish absolute and relative interpretations (see Fitzgibbons, Sharvit & Gajewski 2008 for some discussion).

(34) $\exists_6$ [John climbed $[\![$the highest mountain anyone saw$]\!]^{w,g}$]



Unfortunately, the truth conditions spelled out above are not quite right. They predict that in order for the description to be well-defined, there must be a single person g_6 who saw a mountain higher than any mountain seen by anyone else. But empirically, *the highest mountain anyone saw* could in fact have been seen by many people, so long as it is higher than any other seen mountain. The error here is telling. Part of the predicted felicity conditions for the absolute description are correct: there cannot be ties among the seen mountains; one of them really should be higher than the others. But we have ended up predicting that in addition, there cannot be ties among *the mountain seers*; one of them is incorrectly required to have outseen all the others.

Worryingly, the bad prediction here arises precisely because this is exactly the prediction we want to make for *relative* superlatives. For instance, *the person who climbed the highest mountain was* g_6 really does require that g_6 climbed a higher mountain than any mountain climbed by anyone else. In going so far out of our way to make this absolute description look like a relative one — for the sake of trapping the NPIs in an SDE position — we lose track of what it is we're comparing.

We reluctantly conclude, then, with Howard, that absolute superlatives cannot be derived from est^{H2} in a way that would explain their capacity to license NPIs.

4.2 Relative readings with description-external and non-matching NPIs: Generalization II

The problem of extending est^{H2} to absolute superlatives is not a knock-down argument against it since, as noted above, it is conceivable that the superlative morpheme is ambiguous, and that absolute superlatives are formed with est^{vF} instead. More troubling is the fact that the analysis cannot account for all attested NPIs in relative readings. The examples in (35) provide a variety of problematic cases — some constructed, some lightly edited from text found online — all of which can (or even must) be interpreted relatively despite containing currently unlicensable NPIs.

In particular, the NPIs in (35a–e) are altogether outside of the relative superlative descriptions, whether those descriptions are argumental, as in (35a,b,e,f), or adverbial, as in (35c). And though the NPIs in (35f–i) are inside their superlative descriptions — where we've come to expect them — these descriptions violate Howard's (2014) "matching" criterion. In (35f,g,i), for instance, the description-internal NPIs occur in *arguments* of the noun, not relative clauses that might restrict the superlative morpheme, and even in (35h), the NPI-bearing relative clause has nothing to do with the matrix clause.

Note that all of the examples become ungrammatical if the superlatives are replaced by plain definite or plain adverbial expressions, as in (36). This strongly suggests that it really is the superlative that is responsible for the grammaticality here.[6] Also note that in the presence of description-external NPIs, even those superlatives that are in general ambiguous between absolute and relative interpretations, like *highest* in (35a) and *largest* in (35e), have only relative interpretations in these sentences. If the NPIs are removed or replaced with plain indefinites, absolute readings re-emerge.

(35) a. The judge who gave the highest score to any rookie later regretted it
b. Economics is the field in which the fewest women have ever won a Nobel Prize
c. Which car went the fastest during any 1 second point of the race?
d. John has donated the most money to any third-party candidate
e. The zoo that the largest animal has ever escaped from is in Kansas City, and it was a rhinoceros
f. Which would you say is the company that has the best excuse for any security breach so far?
g. Mary recommended the student who wrote the harshest critique of any book on the syllabus
h. Of all the perturbations examined, PIC treatment at 1 week resulted in the smallest GnRH-1 cells that exhibited any peaks
i. Our goal with this satellite is to capture the best possible image of any asteroid in the Kuiper Belt

(36) a. *The judge who gave the high score to any rookie later regretted it
b. *Economics is the field in which the few women (on stage) have ever won a Nobel Prize
c. *Which car went really fast during any 1 second period of the race?

6 Or really, we should say, the items that were ambiguous between NPI and Free Choice interpretations — e.g., the *any* in (35a) — have only Free Choice interpretations when the superlative is absent.

To be clear about the problem that these sorts of examples pose, look at the subject of (35a), repeated in (37). The right meaning, paraphrased in (37a), could be generated by the LF in (37b), which employs the traditional association-with-focus mechanism introduced in Section 3.1. But as we've seen, the NPI in this LF would not be licensed, since it occurs in the nuclear scope of $[\![\text{est}^{\text{H2}}]\!]^{w,g}(C)$, not the restrictor C.

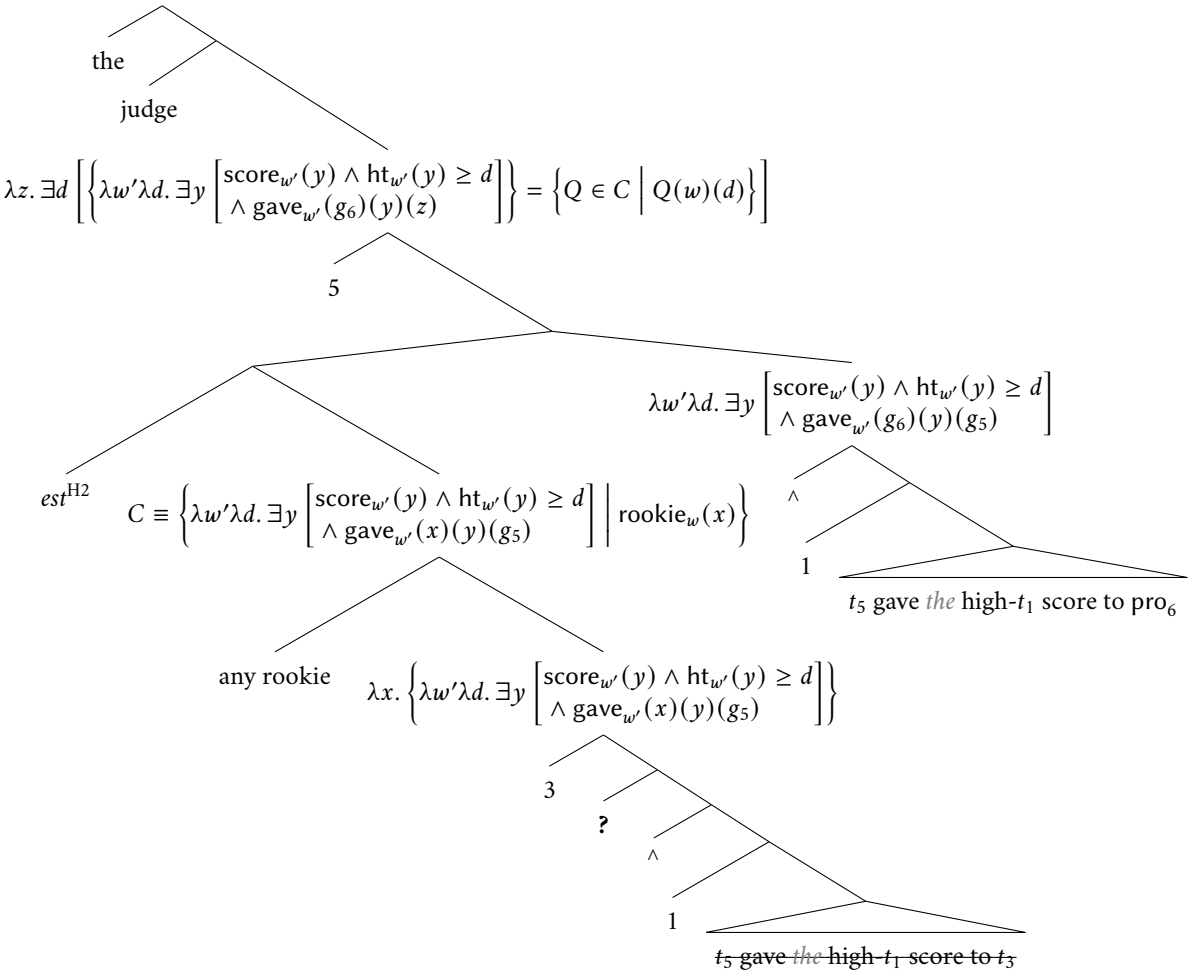
(37) the judge who gave the highest score to any rookie

a. ✓ the judge who gave a higher score to a rookie than any other judge did

b. [the judge [5 [*est*$^{\text{H2}}$-*C* [[^ [1 [[t_5]$_{\mathbf{F}}$ gave *the* high-t_1 score to any rookie]]] ~ *C*]]]]

And there is no obvious well-formed LF in which the NPI occurs in a constituent that would deliver a set of degree properties to the superlative, even with extremely liberal notions about ellipsis. Certainly the NPI doesn't occur in a relative clause that the superlative c-commands, as in Howard's (2014) examples. And even if the entire overt relative clause containing both the superlative and NPI could play this role, say, as in (38), we doubt very much that the correct meaning could be derived. For instance, (38) denotes, for some choice of rookie g_6, the unique judge z who gave g_6 a higher score than they$_z$ gave to any other rookie. This is not what (37a) describes. The problem is that in Howard's cases, the NPI really does constitute the intuitive comparison class, from which the properties the superlative quantifies over are generated. But in (37a), the NPI is incidental to the comparison, which is between judges, not rookies. What's more, how could this ever help with examples like (35d), where the superlative clearly does associate with the (focused) subject, just the same as it does in paradigm cases like *JOHN gave Mary the highest grade*?

(38) $[\![$the judge who gave the highest score to any rookie$]\!]^{w,g}$



So summing up, we have seen that the two most common scope-based approaches to relative readings, est^{H1} and est^{H2}, are both predicted not to license NPIs in places where they do in fact occur. The former has only one explicit argument for the constituent it scopes over, but this argument is not SDE. The latter has been argued to at least sometimes pick up both of its arguments compositionally, one of which is indeed SDE (and not SUE). So in these circumstances, NPIs that occur in that restrictor argument are accounted for. But on the one hand, we found no way to extend this sort of analysis to absolute superlatives, and on the other hand, the range of NPIs actually captured through this loophole does not exhaust their empirical distribution.

In the next section, we therefore approach the problem from the other direction. If popular analyses of relative superlatives cannot be massaged into an adequate account of NPI-licensing, maybe our original analysis of absolute superlatives — which was already SDE exactly where we wanted it to be — can be massaged to derive relative readings.

5 Explaining away the absolute/relative ambiguity

In fact, the literature on superlatives (e.g., Heim 1999, Farkas & Kiss 2000, Sharvit & Stateva 2002) has considered various ways to treat relative readings as special cases of absolute ones. We explore a specific implementation of this idea that keeps est^{vF} as the only superlative morpheme.

5.1 Relative readings via pragmatic domain restriction: no description-external NPIs

Suppose *John climbed the highest mountain* presupposes that there is a mountain that is highest among some contextually relevant set of mountains C. This might be predicted by the following LF, where the domain restriction of the definite article (which has so far been suppressed) is a variable that picks out C.[7]

(39) John climbed the highest mountain

a. John climbed [the-C [[high est^{vF}] mountain]]

Now, there are many ways to be relevant if you're a mountain. For starters, you could just be yourself, since we're talking about mountains. Alternatively, you could belong to some subset of climbed mountains, since we're talking about mountain climbing. Suppose that the readings that we call "absolute" and "relative" are merely reflections of these different choices for the domain of quantification. The freedom to choose different values for C perhaps even avoids the problem that on its relative reading, *John climbed the highest mountain* can be felicitous (and true) when John climbs more than one mountain of the same height; when evaluating the sentence, the hearer may exclude from C some mountains that John climbed, as long as one of John's highest remains.

Yet there are still obvious problems with this approach when it comes to ties. If John and Mary climb the same mountain, and it is higher than any mountain climbed by anyone else, then (39) is false, but it is predicted to be true if defined. If John and Mary climbed distinct mountains of equal height, then the negation of (39) — *John didn't climb the highest mountain (he and Mary were tied)* — is true, but it is predicted to be undefined, unless John's highest mountain is (implausibly) excluded from C. This makes the theory quite weak. Still, the predictions of est^{vF} regarding NPI-licensing, in both absolute and now "relative" readings, are worth exploring. If the approach turns out to be advantageous from the perspective of polarity, it might make sense to seek a pragmatic solution to these definiteness problems.[8]

Let us, then, temporarily ignore the complications with existence and uniqueness, and assume that est^{vF} is the only superlative morpheme in English. Being SDE, $[\![\text{est}^{vF}]\!]^{w,g}(R)$ is predicted to license description-internal NPIs in both absolute and relative scenarios for any R. However, accounting for *description-external* NPIs, and explaining why these obligatorily disambiguate toward relative readings, is not a straightforward task. Consider the case in (40) again.

(40) The judge who gave the highest score to any rookie was Mary

a. ✓ Mary gave a higher score to a rookie than any other judge did

b. the [judge [5 [t_5 [gave [the-C [[high est^{vF}] score]] [to any rookie]]]]]

The NPI *any rookie* plainly does not occur in any argument of est^{vF}, regardless of what domain C is understood to denote. What could be done about this? Since the superlative description in (40b) is necessarily interpreted under a relative-like domain restriction — where C is the set of scores given by judges to rookies — perhaps we should look to the similar relativizing effect that focus is known to produce. The LFs in (41) and (42) exploit the same focus-association mechanism that powers est^{H2} (cf. Heim 1999): [$\bigcup$ C] is the restrictor of *the*, and *the*-[$\bigcup$ C] est^{vF} *grade* is raised to a pre-subject position, forcing C to receive, via association with focus, a value that we identify as "relative".[9]

7 We would also have to assume that the superlative quantification itself is restricted to elements in this domain. Otherwise, we'd predict *[the-C [[high* est^{vF}*] mountain]]* describes Mt. Everest, *if it is in C*, and fails to refer otherwise, rather than predicting that it refers to the highest *of the mountains in C*, which is the goal.

8 There are additional concerns regarding VP-modifying superlatives, as in *Mary sang the loudest*, and amount superlatives, as in *Mary read the most books*. But these could perhaps be analyzed as event-referring (as in Sharvit & Stateva 2002) and degree-referring (as in, e.g., Loccioni 2018 and Wilson 2018), respectively.

9 We use the convention $[\![\bigcup \text{C}]\!]^{w,g} \equiv \bigcup\{\{x \mid P(x)\} \mid P \in C\}$, where, as before $C \equiv [\![\text{C}]\!]^{w,g}$

(41) JOHN gave Mary the highest grade

a. [the-[$\bigcup$ C] [[high *est*^{vF}] grade]] [[2 [John$_F$ gave Mary t_2]] ~ C]

b. $[\![(41a)]\!]$ is defined only when $C \subseteq \{\lambda x.\, \text{gave}_w(\text{m})(x)(z) \mid z \in \mathcal{D}_e\}$, and thus
$[\![\bigcup C]\!]^{w,g} \subseteq \{x \mid \exists z\, [\text{gave}_w(\text{m})(x)(z)]\}$, i.e., things given to Mary

If defined, $[\![(41a)]\!]$ is true iff John gave Mary the highest of the grades that were given to Mary

(42) John gave MARY the highest grade

a. [the-[$\bigcup$ C] [[high *est*^{vF}] grade]] [[2 [John gave Mary$_F$ t_2]] ~ C]

b. $[\![(42a)]\!]$ is defined only when $C \subseteq \{\lambda x.\, \text{gave}_w(y)(x)(\text{j}) \mid y \in \mathcal{D}_e\}$, and thus
$[\![\bigcup C]\!]^{w,g} \subseteq \{x \mid \exists y\, [\text{gave}_w(y)(x)(\text{j})]\}$, i.e., things John gave

If defined, $[\![(42a)]\!]$ is true iff John gave Mary the highest of the grades that he gave

Following this strategy, the relative reading of (40) would include the LF in (43) as part of the derivation of its relative clause. But this hardly helps in explaining the felicity of the description-external NPI, since *any rookie* still does not occur in an argument of the superlative.

(43) t_5 gave the highest score to any rookie

a. [the-[$\bigcup$ C] [[high *est*^{vF}] score]] [[2 [[t_5]$_F$ gave t_2 to any rookie]] ~ C]

b. $[\![(43a)]\!]$ is defined only when $C \subseteq \{\lambda x.\, \exists y\, [\text{rookie}_w(y) \wedge \text{gave}_w(y)(x)(z)] \mid z \in \mathcal{D}_e\}$
$[\![\bigcup C]\!]^{w,g} \subseteq \{x \mid \exists z \exists y\, [\text{rookie}_w(y) \wedge \text{gave}_w(y)(x)(z)]\}$

If defined, $[\![(43a)]\!]$ is true iff John gave a rookie the highest of the scores that were given to a rookie

But notice that now, the meaning of the superlative description is in an odd way entangled with a constituent containing the NPI, such that any variation in the content of the NPI would influence the meaning of the superlative description, by way of its association with focus. One might hope then that we could establish some sort of SDE pattern in the position of the NPI at some constituent containing both the NPI and the entangled superlative. For instance, we might expect (44) — the function represented by abstracting over the grammatical NPI's position in (43) — to be SDE. This would be in line with theories that predict NPI acceptability by testing whether one syntactic constituent is SDE with respect to a larger constituent containing it, rather than by testing the monotonicity of a particular operator that scopes over it (Heim 1984, Gajewski 2005, Homer 2008).

(44) $[\![9$ [[the-[$\bigcup$ C] [[high *est*^{vF}] score]] [[2 [Mary$_F$ gave t_2 to t_9]] ~ C]]$]\!]^{w,g}$

But this is a fraught question. Different values for t_9 — $[\![$a rookie$]\!]$, $[\![$a player$]\!]$, $[\![$a person$]\!]$, etc. — will lead to different (usually incompatible) constraints on C, thanks to the presuppositions imposed by focus. The focus antecedent C does not vary with the value of t_9. In any given context, it will be fixed, even as different arguments are passed into (44). So when (44) is applied to $[\![$a rookie$]\!]$, every property in C will have to pick out, for some $z \in \mathcal{D}_e$, the complete set of scores that z gave to rookies. But when (44) is applied to $[\![$a player$]\!]$, every property in C will have to pick out, for some $z \in \mathcal{D}_e$, the complete set of scores that z gave to players. So in any context where judges all gave at least one non-rookie a score they didn't give to any rookie, C would have to be empty in order for both functions to be well-defined. But then of course the superlative description itself will be undefined, since there won't be anything in the definite's domain restriction.[10]

So for a pattern like this to be the explanation for the acceptability of (40), the licensing criterion on NPIs would have to be relaxed even further. Perhaps it would be enough simply for (45a) to Strawson entail (45b), but not vice versa (given that $[\![\text{rookie}]\!]^{w,g} \Rightarrow [\![\text{player}]\!]^{w,g}$). Note that the superlative in (45a) associates with focus in the usual way, but the superlative in (45b) receives its domain restriction $\bigcup$ C merely from the context of the

10 See also von Fintel 1999: pp.134–135, who points out a similar technical hiccup in the examination of adverbial *only*'s monotonicity.

monotonicity test itself. The C is identified with whatever set of properties is taken to establish the (relative) comparison class in (45a).

(45) a. [the-[$\bigcup$ C] [[high *est*^{vF}] score]] [[2 [Mary$_{\mathbf{F}}$ gave t_2 to any player]] ~ C]
b. [the-[$\bigcup$ C] [[high *est*^{vF}] score]] [2 [Mary$_{\mathbf{F}}$ gave t_2 to any rookie]]

But even this test fails. The LF in (45a) is true when (i) $[\![\bigcup \text{C}]\!]^{w,g}$ is a set of scores given to players, (ii) there is a single score in $[\![\bigcup \text{C}]\!]^{w,g}$ higher than all the others, and (iii) Mary gave this score to some player. The definite/superlative presuppositions of (45b) add nothing to this, since they are already factored into the truth conditions of (45a). The focus marking itself in (45b) perhaps contributes a presupposition that someone gave this score to a rookie. But still, all of this together does not entail (45b); Mary might have given this highest score to some non-rookie player while someone other than Mary gave it to a rookie. And note that switching which of the two LFs the comparison class is associated with doesn't help. If $[\![\bigcup \text{C}]\!]^{w,g}$ is the set of scores assigned to rookies rather than players, then (45a) is true when Mary gave some player the highest score given to any rookie. This again does not entail that Mary gave some *rookie* that score (though somebody must have).

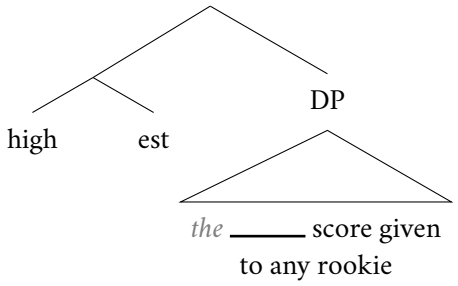
Not that any of it matters, because for absolutely any domain restriction on the definite/superlative, (45b) is going to (classically!) entail (45a). The superlative description in both LFs, if it is well-defined, will refer to the same object, say s*. If the latter LF is true, then Mary gave s* to a rookie, and therefore, to a player, so the former LF is true. So the position of the NPI couldn't possibly be SDE *without also being SUE*. Presumably this point extends to *any* analysis of relative readings that does not give the superlative description-external scope. If the superlative is really contained in its own definite description, then no matter what object that description refers to, the superlative will be powerless to influence the monotonicity of positions elsewhere in the sentence.

This brings us to the conclusion that classic unifications of absolute and relative readings under the umbrella of domain restriction are in no position to account for the general pattern of NPI-licensing in relative readings. We are led then to seek an alternative analysis of relativity, one that gives the superlative genuine scope over all of the syntactic material that influences its interpretation, but nevertheless treats that material as if it were part of its restriction or comparison class, rather than part of its measure. The next section presents one such analysis.

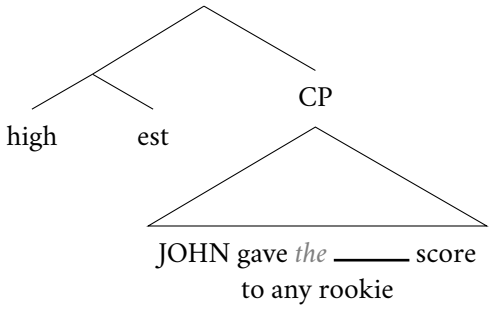
6 Superlatives as quantifiers on sets of assignments: Bumford 2017a

The polarity data paint the following picture:

(46) a. the highest score given to any rookie in the competition



b. JOHN gave the highest score to any rookie in the competition



Ignoring the definite article for a moment, we've seen that the classic, *in situ*, absolute superlative *est*^{vF} takes two arguments, a measure function R and a comparison class of things to measure P. In (46a), for instance, R is the denotation of the adjective, $[\![\text{high}]\!]$, and P is the denotation of the rest of the noun phrase, the set of scores given to rookies. The comparison class argument is SDE (but not SUE). The wide distribution of

acceptable NPIs around *relative* superlatives suggests that their LFs ought to look similar. In (46b), for instance, the first argument of the superlative should again be a measure function determined by the gradable adjective, ⟦high⟧, and everything else should be contained in a second argument that behaves like *P*, an SDE (but not SUE) comparison class.

Unfortunately, in (46b) the superlative's second argument cannot simply denote exactly what the DP of (46a) denotes — the set of scores given to rookies — because the result of evaluating (46b) should not be a score, but rather a proposition that is true iff John was the most generous rookie-scorer. We would do a little better to have the CP denote a relation between judges and the scores they assigned to rookies. Then the superlative might be reanalyzed to use focus-marking to ensure that John is the argmax of this relation. But this would strain any sense of uniformity between absolute and relative readings, and certainly the sense of a unified account of NPI licensing, since in absolute derivations the NPIs would be in a genuine comparison class but in relative derivations the NPIs would be in some sort of "association relation" (Farkas & Kiss 2000). In any case, this relation argument would not be SDE (see Appendix B).

We are aware, though, of one analysis of superlatives that has the desired shape. Bumford (2017a,b) offers a dynamic account of superlative operators, where adjectives like *highest* denote filters on sets of satisfying assignment functions, and can be applied uniformly to sets of individuals that might witness the referent of a description, as in (46a), and sets of propositions that might witness the truth of a sentence, as in (46b). We first present a simplified version of this analysis, and then prove that it has the desired monotonicity properties.

6.1 A simple dynamic semantics

As a bridge between the sorts of static analyses of superlatives in Heim 1999, von Fintel 1999, and Howard 2014 presented so far and the dynamic analysis on offer in Bumford 2017a, we introduce two object-language operators in (47). The first simply stores the current assignment together with the value of its complement in a singleton set (cf. Karttunen's (1977) ⟦?⟧ and Partee's (1986) IDENT, both used in Section 4). The second is effectively a "nondeterministic" analog of the Heim & Kratzer (1998) abstraction index: given a *set* of possible values *G*, it shifts the assignment at which its complement β is evaluated once for each element of *G*. For comparison, the traditional abstractor is shown in (47c). Both of these binders will be assumed to adjoin to the sisters of moved DPs, as is standard, with the choice between them determined by the semantic type of the DP; set-denoting DPs will require $\star_n$, others the plain n.

(47) a. $[\![\eta]\!]^g := \lambda a.\, \{\langle a, g\rangle\}$

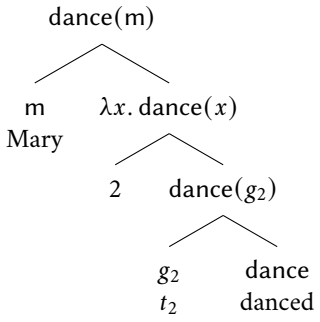
b. $[\![\star_n\, \beta]\!]^g := \lambda G.\, \bigcup \left\{ [\![\beta]\!]^{h^{n \mapsto a}} \;\middle|\; \langle a, h\rangle \in G \right\}$

c. $[\![n\, \beta]\!]^g := \lambda a.\, [\![\beta]\!]^{g^{n \mapsto a}}$

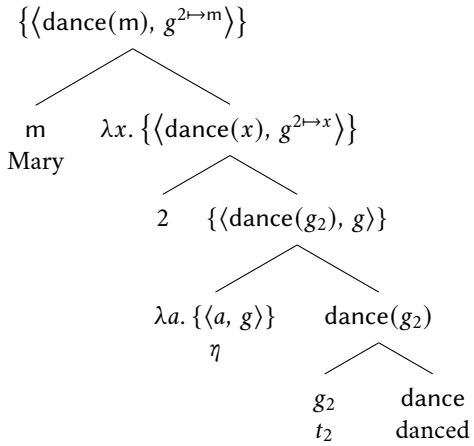
This modular formulation of dynamic techniques loosely follows Charlow 2019. There are "static" derivations in which binding information is only part of the input to the evaluation function ($[\![\cdot]\!]^g$), alongside "dynamic" derivations in which binding information is also part of the output of the evaluation function ($[\![\cdot]\!]^g = \{\langle \alpha, g'\rangle \mid \cdots\}$). This is illustrated in (48).[11] In annotating these trees with denotations, there is the usual awkwardness of having to choose a concrete assignment to display for nodes in the scope of a binding index, even though this binder guarantees that the node will be evaluated at many different assignments. We choose to describe the denotation of every constituent as if it were evaluated at the formal parameter g.

11 The fragment is extensional from here on out, so we suppress world parameters in what follows.

(48) a. $[\![\text{Mary danced}]\!]^g$



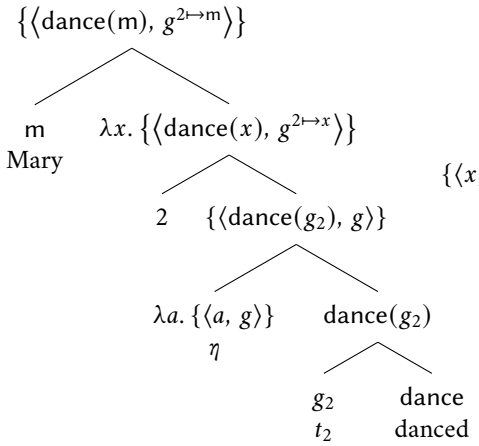
b. $[\![\text{Mary danced}]\!]^g$



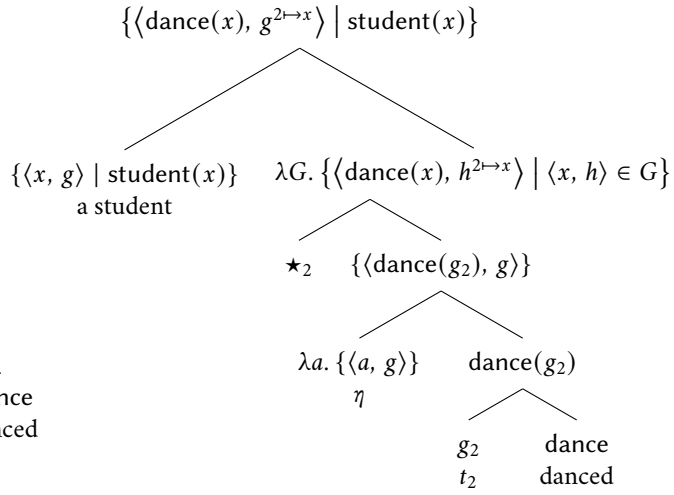
To this we add the nondeterministic indefinite defined in (49).[12] Note that $[\![\text{a NP}]\!]^g$ is essentially the characteristic set of the restrictor NP, except that each potential witness x is paired with an assignment that witnesses the truth of evaluating $[\![\text{NP}]\!]^g$ at x. Together with the $\star$ abstraction operator, this allows all of the possible referents for the indefinite to modify the current assignment "in parallel". This is shown in (50b), where (50a) repeats (48b) for comparison.

(49) $[\![\text{a}]\!]^g := \lambda P.\, \{\langle x, h\rangle \mid \langle \mathbf{1}_t, h\rangle \in P(x)\}$

(50) a. $[\![\text{Mary danced}]\!]^g$



b. $[\![\text{A student danced}]\!]^g$

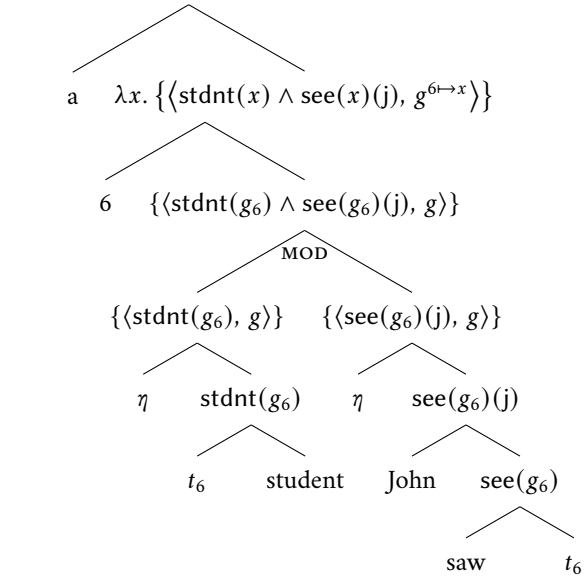


To simplify derivations of complex DPs, we follow Heim 1997 in allowing a determiner to bind the "subject" position of its nominal as well as the gap in that nominal's relative clause. Schematically: [$_{\text{DP}}$ Det n [$_{\text{NP}}$ [t_n Noun] [$_{\text{CP}}$ who … t_n …]]]. This is in no way essential, and derivations can be mechanically converted to the format in Heim & Kratzer 1998. The LFs in (51) provide a template. In (51a), a single abstraction index binds the argument position of 'student' as well as the object of 'saw' in the relative clause. The MOD mode of combination is simply (dynamic) conjunction: $[\![\,[\alpha\ \beta]\,]\!]^g := \left\{\langle p \wedge q, k\rangle \;\middle|\; \langle p, j\rangle \in [\![\alpha]\!]^g, \langle q, k\rangle \in [\![\beta]\!]^j\right\}$. In (51b), a vacuous subject PRO binds the argument position of 'student', and an implicit relative pronoun binds the object of 'saw' (cf. Heim & Kratzer 1998: Ch. 8.5). The MOD mode of combination is (dynamic) predicate conjunction: $[\![\,[\alpha\ \beta]\,]\!]^g := \lambda x.\, \left\{\langle p \wedge q, k\rangle \;\middle|\; \langle p, j\rangle \in [\![\alpha]\!]^g(x), \langle q, k\rangle \in [\![\beta]\!]^j(x)\right\}$.

12 We write $\mathbf{1}_t$ for the model-theoretic object representing truth.

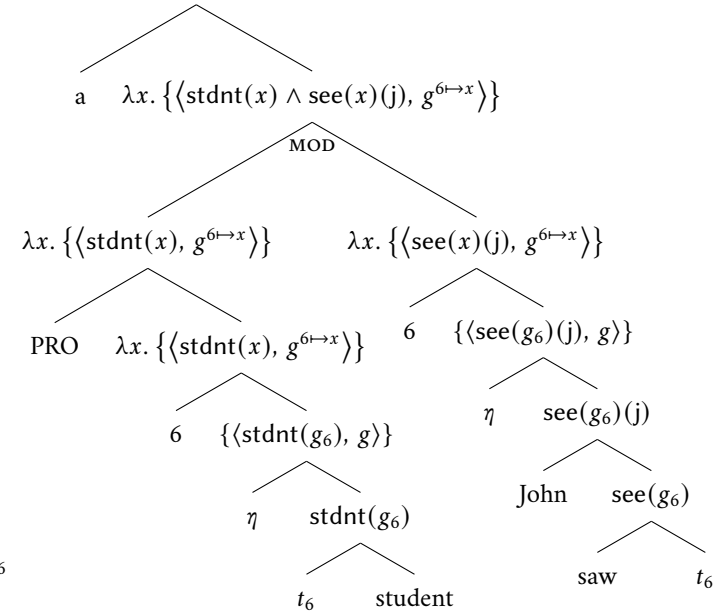
(51) a. $[\![\text{a student John saw}]\!]^g$

$\{\langle x, g^{6\mapsto x}\rangle \mid \text{stdnt}(x) \wedge \text{see}(x)(\text{j})\}$



b. $[\![\text{a student John saw}]\!]^g$

$\{\langle x, g^{6\mapsto x}\rangle \mid \text{stdnt}(x) \wedge \text{see}(x)(\text{j})\}$



6.2 Superlatives as dynamic operators

Building on a compositional semantics along these lines, Bumford (2017a) hypothesizes that superlative adjectives filter sets of possible *evaluation outputs* — value-assignment pairs — by measuring them along the coordinate that their DP is associated with. As shown in (52a), the denotation of the superlative morpheme itself takes a comparative relation as argument,[13] and returns a function from sets of outputs to sets of outputs, sometimes called a postsuposition (Brasoveanu 2013). For instance, $[\![\text{highest}_n]\!]^g$ surveys a set of results and keeps only those whose assignments are undominated in the height of the value they assign to n.

(52) a. $[\![\text{-est}_n]\!]^g := \lambda R \lambda G. \left\{\langle a, h\rangle \in G \;\middle|\; \neg\exists\langle b, h'\rangle \in G \left[R(h_n)(h'_n)\right]\right\}$

b. $[\![\text{higher}]\!]^g := \lambda y \lambda x. \text{ht}(x) > \text{ht}(y)$

c. $[\![\text{highest}_n]\!]^g := [\![\text{-est}_n]\!]^g([\![\text{higher}]\!]^g)$

$= \lambda G. \left\{\langle a, h\rangle \in G \;\middle|\; \neg\exists\langle b, h'\rangle \in G \left[\text{ht}(h'_n) > \text{ht}(h_n)\right]\right\}$

To avoid some of the complications mentioned earlier regarding the definite determiner in superlative descriptions, we make two simplifying assumptions in this section:

(i) Use of a superlative adjective is only felicitous if its comparison class argument G initially contains several distinct values (usually at least three), but only one of these remains after non-dominant outputs have been removed. In other words, there should be at least a few candidates, but only one winner. Formally, for any input g and set G such that $\left|\{a \mid \langle a, h\rangle \in G\}\right| > 1$, it must be true that $\left|\{a \mid \langle a, h\rangle \in [\![\text{highest}_n]\!]^g(G)\}\right| = 1$.

(ii) As in the classic scope-theoretic accounts (Szabolcsi 1986, Heim 1999), the definite determiner of a superlative description is interpreted as if it were in fact indefinite. In this dynamic setting, that means that it introduces a set of potential witnesses for its restrictor, and pairs each one with an independently modifiable assignment, exactly like $[\![\text{a}]\!]^g$.

13 Here we are following Stateva 2002 and Bobaljik 2012 in assuming that *highest* is underlyingly *[[high -er] -est]*, but nothing hinges on this; (52a) may be trivially adjusted to accept a gradable adjective of type (e, d) or $(e, (d, t))$ directly.

These assumptions are obviously connected, and a more robust analysis would presumably seek to derive the uniqueness of superlative descriptions from the definite article that they co-occur with. But since issues of definiteness in relative readings are well-known and much-discussed (Szabolcsi 1986, Heim 1999, Sharvit & Stateva 2002, Coppock & Beaver 2014), and since our chief concern here is monotonicity, we opt for the blunt approach of Heim (1999). The reader is referred to Bumford 2017a for an extended attempt to parcel out the respective compositional contributions of *the* and *-est*.

Finally, we assume that superlative adjectives may take scope over any constituent where it would be well-typed and felicitous for them to do so.[14] Perhaps unexpectedly, the adjectives in the LFs below do not leave traces, and so do not abstract over their initial positions. This again is motivated by the types. The situation tends to arise when semantic modifiers are analyzed as propositional operators (see, for instance, von Fintel 1997 on *only*, or Crnič 2014 on *even*). This is because the initial position of a modifier is by definition not an argument position. There is no need to bind a variable there. The neat thing about this particular formulation of dynamic semantics is that DPs and CPs have essentially the same type; they both denote sets of pairs of values with assignments. So even when the adjective modifies its local DP, it acts on that DP the same way a propositional operator would act on a sentence, as desired in (46).

But here, too, this is not essential. Along the lines of Barker's (2007) treatment of 'same', the superlative could abstract over a property — type (e,t) — or a property-modifier — type $((e,t),(e,t))$ — when it raises. It would then have to fill this hole with a trivial universal property, or an identity function, as illustrated in (53a). Alternatively, one could scope the superlative together with its definite article as a kind of "complex quantifier", abstracting over the type of a determiner, as in (53b). This, in fact, would be quite similar to Bumford's (2017b) original analysis (see also Szabolcsi 1986 on 'the -est', Zimmermann 2003 on 'the occasional', Kennedy & Stanley 2009 on 'the average', and Morzycki 2016 for a survey of related constructions and analytical prospects).

(53) Alternative, trace-binding analyses superlative adjectives (not pursued here)

a. $[\![\text{highest}_n]\!]^g := \lambda F.\left\{\langle a,\, h\rangle \in F(\lambda P.\, P) \;\middle|\; \neg\exists\langle b,\, h'\rangle \in F(\lambda P.\, P)\left[\text{ht}\left(h'_n\right) > \text{ht}(h_n)\right]\right\}$
as in, e.g., [highest$_n$ 7 [*the* 2 [η [t_2 [t_7 mountain]]]]]

b. $[\![\text{the-highest}_n]\!]^g := \lambda F.\left\{\langle a,\, h\rangle \in F\left([\![\text{a}]\!]^g\right) \;\middle|\; \neg\exists\langle b,\, h'\rangle \in F\left([\![\text{a}]\!]^g\right)\left[\text{ht}\left(h'_n\right) > \text{ht}(h_n)\right]\right\}$
as in, e.g., [the-highest$_n$ 7 [t_7 2 [η [t_2 mountain]]]]

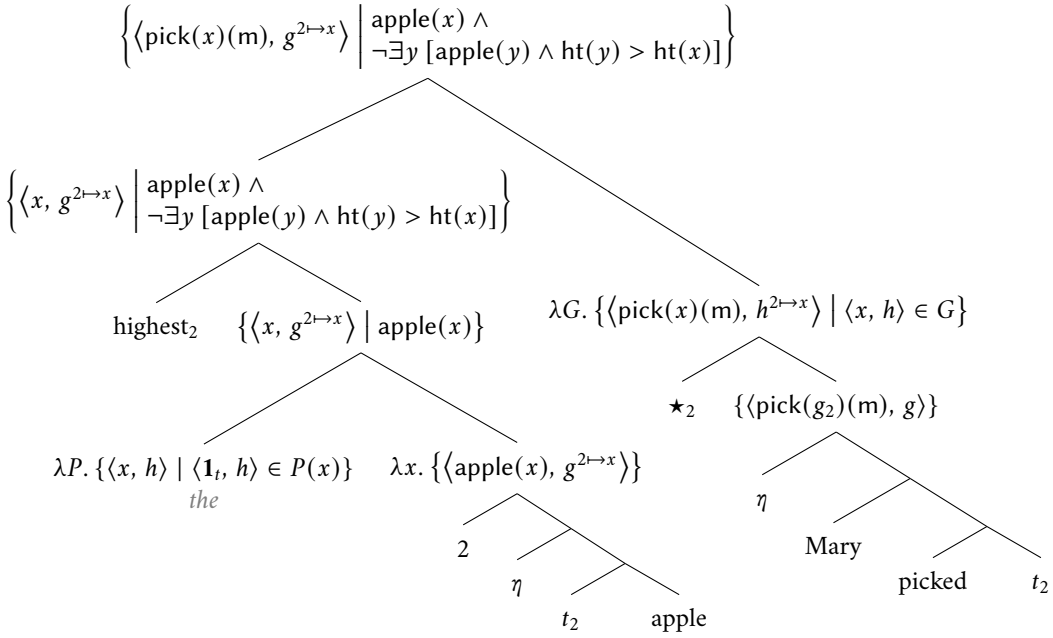
In any case, we do not mean to commit to this scoping of the adjective as a particular form of *movement*, per se. Of course such movement would be quite unorthodox syntactically (though not more unorthodox than the standard Heimian analysis, which raises *just the superlative suffix* out of various left branches). The LFs are intended to show where the various semantic operators take scope, to be worked out using whatever mechanisms for scope-taking are preferred. Bumford's (2017b) version, for instance, relies exclusively on continuation-passing and type-shifting to regulate the order of operations. But again for concreteness, we take it that a superlative adjective may raise at LF, tracelessly, to adjoin to any constituent that dominates it, as long as its type fits and its felicity conditions are met.

Thus we derive absolute superlative descriptions as in (54). The superlative adjective *highest* takes scope over its containing DP. That description serves up a set of potential referents for the direct object, each paired with a modified assignment mapping the index 2 to a student. The superlative eliminates all but the highest such potential referents. Given the assumptions above, there should be only one value x left in play after this

14 In this paper we do not wade into the issue of scope-island constraints on superlative movement, which we see as orthogonal to questions about the monotonicity of the denotation. Szabolcsi (1986), Heim (1999), and many others assume that definite determiners hosting superlative descriptions do not constitute scope islands because such determiners behave as if they are truth-conditionally indefinite in such contexts. See the references in the previous paragraph for more on this issue. Szabolcsi (1986) claims that tensed clause boundaries delimit superlative scope, but only documents attitude reports; Heim (1985: p. 26, ex. (48)–(51)) provides examples of various degree operators, including superlatives, appearing to scope out of tensed relative clauses. We think the matter is quite unclear empirically and so make no stipulations here.

operation. The whole superlative description then takes scope over the sentence, returning the (singleton) proposition that is true iff Mary picked the unique apple that is not below any others.

(54) $[\![\text{Mary picked the highest apple}]\!]^g$



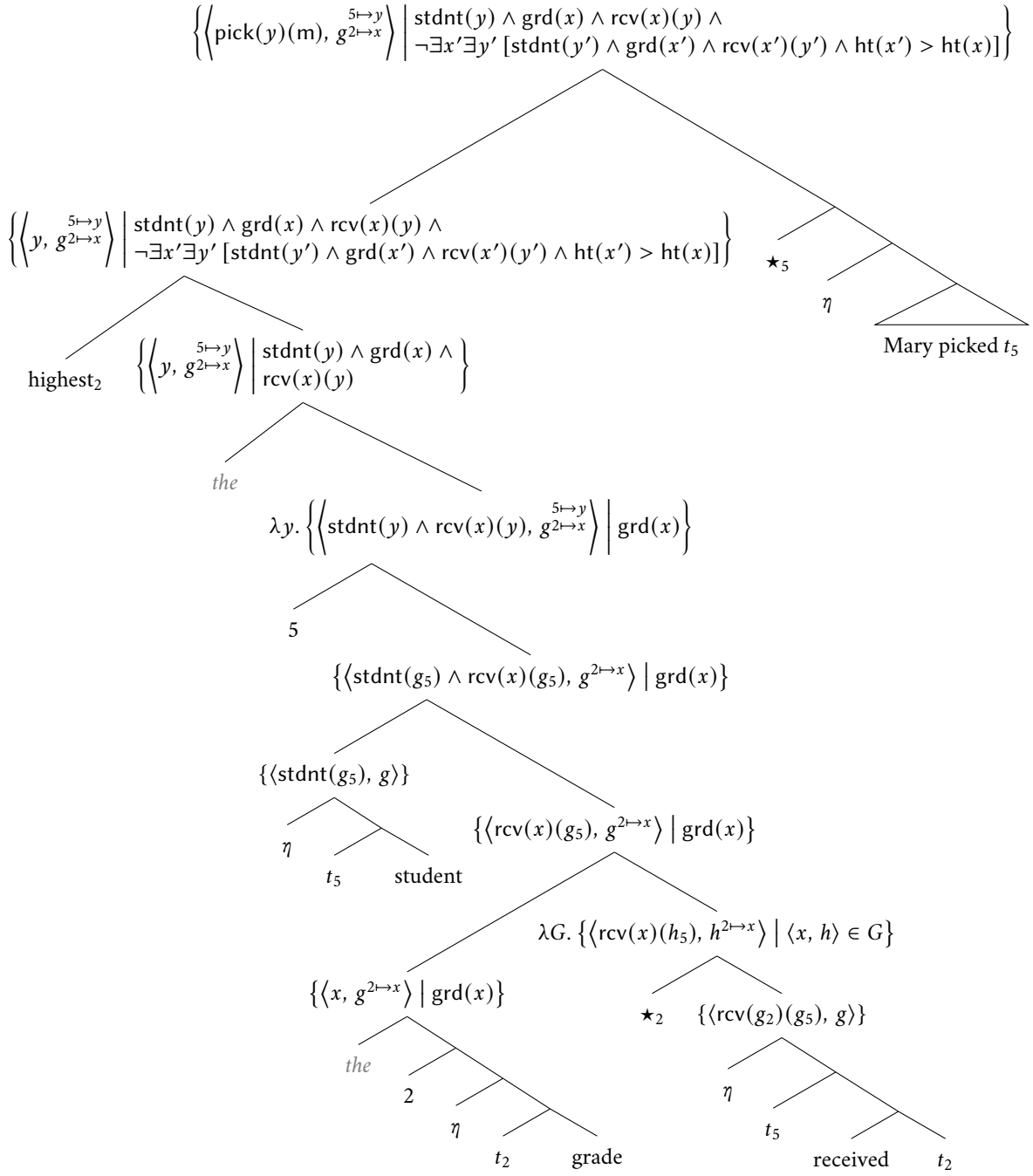
Two things are worth pointing out about this LF and those that follow. First, we are again following Heim 1997 in abstracting over the same index in the description's restrictor and its scope, in this case 2. Of course it would not change the truth conditions to pick different indices for the two abstracts, though it would have the knock-on effect of storing a description's referent at two different locations in the output assignment. We can't see how there could be any harm in this, but we avoid it for perspicuity. Second, the superlative adjective is indexed to the referent introduced by its own DP. Intuitively, nothing else would make sense. In (54), 'highest' should compare assignments along the coordinate that stores apples. But formally, this must be stipulated: we assume 'Det Adj-est NP' is necessarily parsed as [Det n [… [est_n Adj] … NP]]. This is in some sense just the obverse of the normal requirement that a raised element's abstractor should match the index of its local position; here, the raised element's index should match the abstractor above its local position. In these compositional dynamic trees, as in real ones, information flows both up and down the trunk.

Relative readings result when a superlative adjective takes scope over a constituent that further restricts the set of entities that might witness its description. As mentioned earlier, we assume that superlative scope is free within the confines of well-typedness. However, since it must modify a set of *entity*-valued outputs, the opportunities for mobilization are fairly limited. Bumford (2017b) describes two configurations in which this can happen.[15] The first is when a superlative DP is itself contained within another definite description. The second is when a superlative associates with focus. In both cases, the superlative DP's referent becomes dynamically co-dependent on that of another argument in the sentence, either the definite description that hosts it or the focused expression it associates with. Take (55), for example.[16]

15 There are other sources of relative readings documented in the literature, including matrix *wh*-questions and some control constructions. The former are confounded for our purposes because *wh*-questions often license NPIs on their own, and those analyses that deal with the latter typically just resort to formally focus-marking the controlled PRO (e.g., Romero 2013). We leave these for future work.

16 NB: The tree depicts (55)'s relative reading, on which it describes the student who outscored all the others. If *highest* instead scoped only over its own DP — *the grade* — then the reading would be absolute, describing the student who received the max possible score.

(55) $[\![$Mary picked the student who received the highest grade$]\!]^g$



At the edge of the composition of the outer description, all of the potential assignments g under which the sentence could be true have the following property: the student g_5 received the grade g_2. Thus students who did not receive a grade and grades that no students received cannot possibly serve as the witnesses for these respective descriptions, since no assignment g could have values for both g_5 and g_2 that meet this constraint. As a result, such initially available candidate referents fall out of the computation before the superlative adjective is evaluated. Of the remaining potential assignments, the superlative removes all except those that send 2 to a grade higher than any other *remaining candidate* grade. This guarantees that only those students whose grade was at least as high as that of any others are available to serve as the referent of the entire host description. And

again given the assumption above — that the superlative is felicitous only if its result contains a single value across potential outcomes — there should be just one such "winning" student.

This presentation of Bumford's (2017a) analysis suffices to establish the requisite montonicity properties and to verify the predictions against the data introduced above. We return to association with focus in Section 6.4. Recall that the troubling NPIs for most analyses are of the sort in (56), extracted from the sentences in (35).

(56) a. the judge who gave the highest score to any rookie
b. the field in which the fewest women have ever won a Nobel Prize
c. the company that has the best excuse for any security breach so far
d. the zoo that the largest animal has ever escaped from

All of these descriptions involve unambiguously relative readings of superlatives; (56a) describes the judge who gave a higher score to a rookie than any other judge gave to a rookie, (56a) describes the field that has awarded fewer women the Nobel Prize than any other field has, etc. And in each example, an NPI occurs in the semantic scope of the relative superlative (which is the entire host description), sometimes in the superlative's nominal complement (56c) and sometimes not (56a,b,d). Since the containing DPs are all singular definite descriptions, they cannot be the source whatever negativity these NPIs are swimming in. That leaves only the superlative adjective itself. We now establish that any such adjective defined as in (52) is indeed SDE (and not SUE).

6.3 The dynamic superlative is SDE but not SUE

Consider an arbitrary superlative function S_n in (57c). Different underlying comparative relations R will determine different measure functions μ and orderings $>$, but since these do not affect the proof, we gloss over them.

(57) a. $[\![\text{-est}_n]\!]^g := \lambda R \lambda G \lambda \langle a,\, h\rangle\colon G(\langle a,\, h\rangle)$
$\quad . \neg\exists \langle b,\, h'\rangle \left[G(\langle b,\, h'\rangle) \wedge R(h_n)\left(h'_n\right)\right]$

b. $R := \lambda y \lambda x.\, \mu(x) > \mu(y)$

c. $S_n := [\![\text{-est}_n]\!]^g(R)$
$= \lambda R \lambda G \lambda \langle a,\, h\rangle\colon G(\langle a,\, h\rangle)$
$\quad . \neg\exists \langle b,\, h'\rangle \left[G(\langle b,\, h'\rangle) \wedge \mu\left(h'_n\right) > \mu(h_n)\right]$

The only difference between a denotation like (52c) and (57c) is that where earlier, *highest* was taken to denote a (total) function from sets of assignments to subsets of those assignments, here we define S in terms of the sets' characteristic functions. So S is a function from properties of inputs to properties of outputs. Moreover, S is partialized so that it is only defined for arguments that have the property picked out by its comparison class G. This is exactly analogous to von Fintel's (1999) insight that *tallest volleyball player* is a partial function defined only for entities that have the property picked out by *volleyball player*. The only difference is that the properties S applies to are dynamic, in the sense that they are tagged with assignments.

Let G and H be potential arguments of S_n such that $G \Rightarrow H$. And let $\langle \alpha,\, \gamma\rangle$ be in the domain of both $S_n(G)$ and $S_n(H)$. To show that S_n is SDE, we need to show that $S_n(H) \overset{\text{ST}}{\Rightarrow} S_n(G)$, i.e., if $S_n(H)(\langle \alpha,\, \gamma\rangle)$ is true, then so is $S_n(G)(\langle \alpha,\, \gamma\rangle)$.

1. Assume $S_n(H)(\langle \alpha,\, \gamma\rangle)$ is true.

2. By definition of S_n, then, $\neg\exists \langle b,\, h'\rangle.\, H(\langle b,\, h'\rangle) \wedge \mu\left(h'_n\right) > \mu(\gamma_n)$.

3. Assume for contradiction that there is a pair $\langle b^*,\, h^*\rangle$ such that $G(\langle b^*,\, h^*\rangle) \wedge \mu\left(h^*_n\right) > \mu(\gamma_n)$.

4. Then since $G \Rightarrow H$, it should be true that $H(\langle b^*, h^* \rangle)$ and therefore that $H(\langle b^*, h^* \rangle) \wedge \mu(h_n^*) > \mu(\gamma_n)$.

5. But this contradicts Step 2, so in fact Step 3 must be false: $\neg\exists\langle b, h\rangle.\, G(\langle b, h\rangle) \wedge \mu(h_n) > \mu(\gamma_n)$.

6. By definition of S_n, then, $S_n(G)(\langle \alpha, \gamma \rangle)$ is true.

7. Thus, $S_n(H) \overset{\text{ST}}{\Rightarrow} S_n(G)$, completing the proof.

Finally, S_n is not SUE. To see this, let μ be the function that maps letters to their positions in the alphabet, so that S_n is something like "alphabetically earliest". Then let G be the characteristic function of $\{\langle x, [n \mapsto x]\rangle \mid x \in \{\text{'a'},\text{'b'}\}\}$, and let H be the characteristic function of $\{\langle x, [n \mapsto x]\rangle \mid x \in \{\text{'a'},\text{'b'},\text{'c'}\}\}$. Clearly $G \Rightarrow H$, but $S_n(H) = \{\langle \text{'c'}, [n \mapsto \text{'c'}]\rangle\} \not\subseteq \{\langle \text{'b'}, [2 \mapsto \text{'b'}]\rangle\} = S_n(G)$. So $S_n(H) \overset{\text{ST}}{\not\Rightarrow} S_n(G)$, and thus S_n is not SUE.

6.4 Focus as witness-set restriction and identification

Returning to the derivation in (55) for a second, we can see that the outer description — the one specifying a student — plays a crucial role. In dynamic semantics, the assignments paired with values keep track (incrementally) of the potential witnesses for various descriptions. An assignment cannot fail to assign a value to a discourse referent (here, an index). And the values assigned to the various indices have to satisfy the descriptive constraints of the various noun phrases that introduce them. That means that every assignment still in play at the top of the outer description makes a choice among the students and a choice among the grades *such that the one received the other*. So students who were not graded, and grades that no student received, are not among the candidate witnesses. This is what leads to the relative, rather than absolute, comparison of heights.

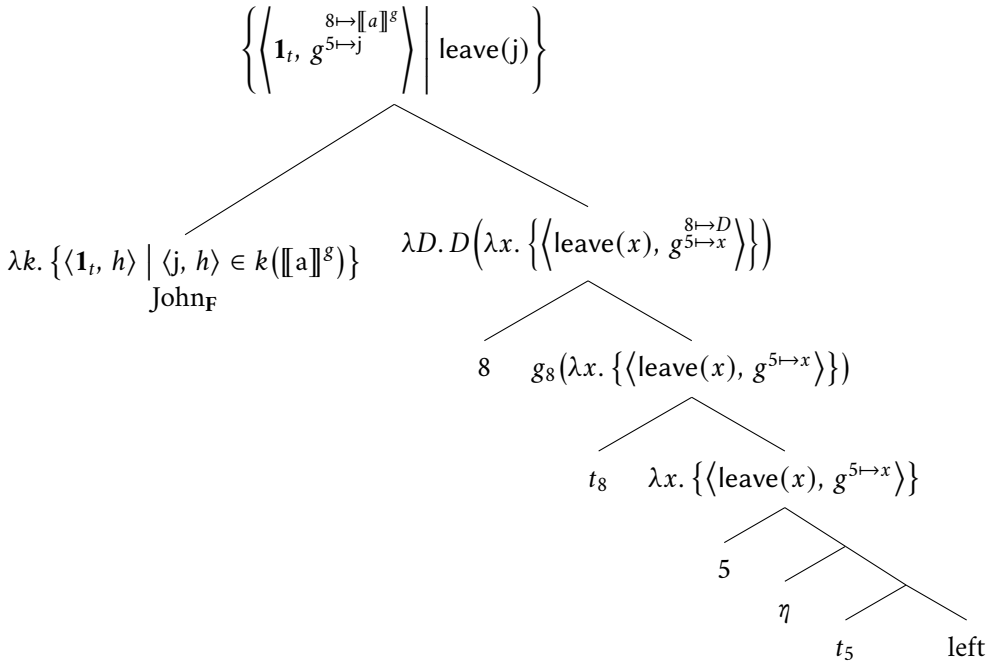
If focus is to have the same relativizing effect, then it too must introduce a discourse referent (again, the way we've set things up, this is just an abstraction index) that in turn imposes relational constraints on the superlative description's own potential witnesses. But a focused expression should do more than just introduce a referent. After all, *JOHN left* does not mean the same thing as *a person left*. So in addition to the variation it introduces, it should also require that its own value be among those witnesses that make the sentence true. Here is a way of caching this out (see Bumford 2017b for a more robust variant, and independent arguments for this approach to focus).

(58) $[\![\cdot_{\mathbf{F}}]\!]^g := \lambda x \lambda k.\, \{\langle \mathbf{1}_t, h\rangle \mid \langle x, h\rangle \in k([\![\text{a}]\!]^g)\}$

This denotation acts locally like an indefinite — $k([\![\text{a}]\!]^g)$ — but then ultimately only returns true — $\langle \mathbf{1}_t, h\rangle$ — if the value of the focused expression is among the potential witnesses for the description — $\langle x, h\rangle \in k([\![\text{a}]\!]^g)$. In other words, *John*$_{\boldsymbol{F}}$ *left* computes whether or not John is among the leavers, as demonstrated in (59). If so, then any constraints on other referents picked up along the way are preserved in h. The denotation at the top of (59) is empty if John didn't leave, which in dynamic semantics amounts to falsity.[17] If John did leave, then there is just one output, representing that the sentence is true, paired with a sole assignment representing John as a potential witness for 5.

17 Formally, φ is "true" in a context g iff there's at least one assignment paired with $\mathbf{1}_t$ among the outputs of φ when evaluated at g. That is, iff there's at least one way of choosing referents for all the descriptions that satisfy all of the predicates that relate them.

(59) $[\![\text{JOHN left}]\!]^g$



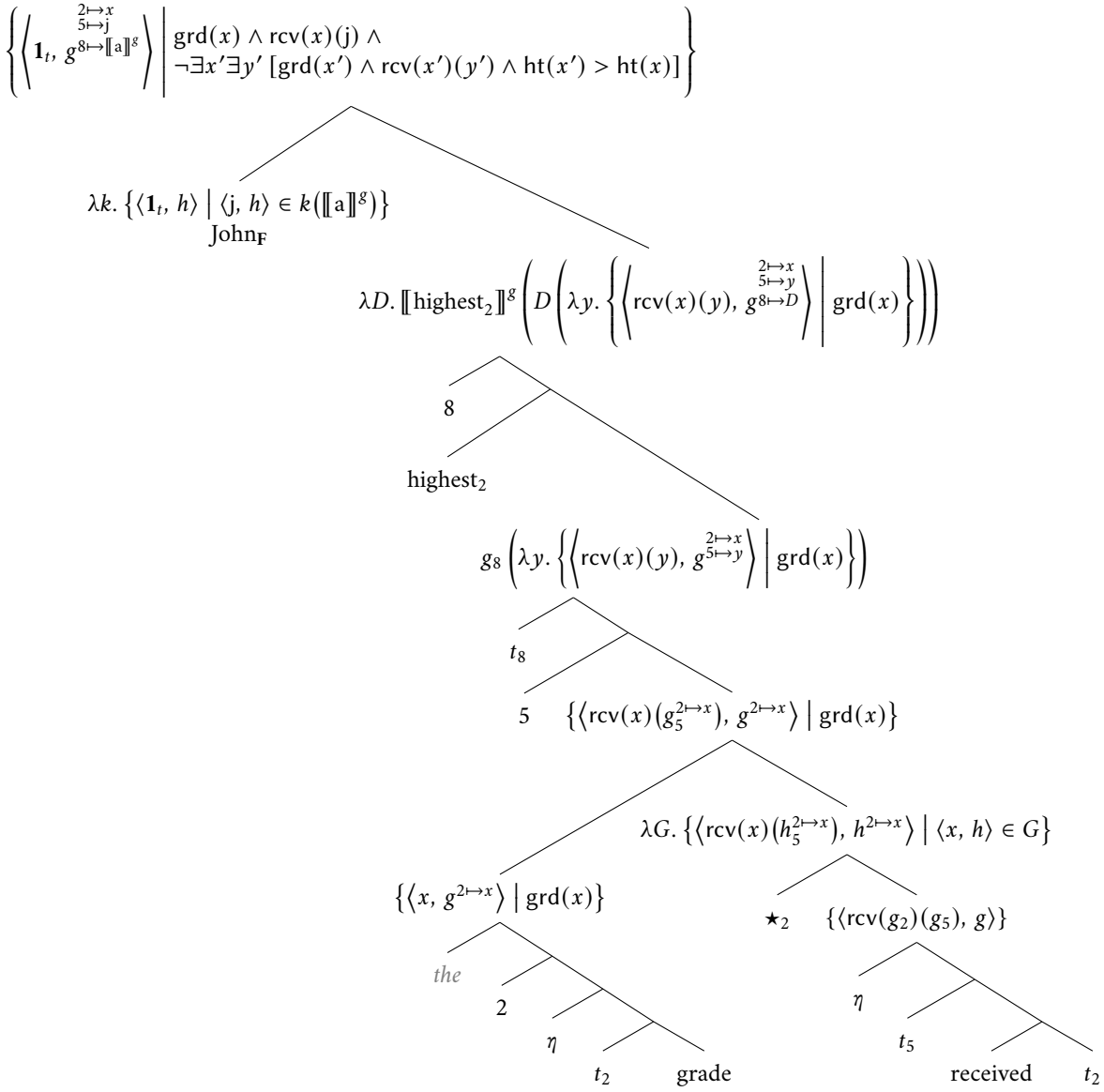
Superlatives associate with focus when they scope in between the temporary indefinite that the focus introduces and the test that is run to ensure that the focused expression survives the update. This is shown in (60). The local position of the focus, $\textit{John}_{\boldsymbol{F}}$, acts indefinitely. It introduces the set of entities y that would render the rest of the LF true, and pairs each of these entities with a grade x that would witness that truth, i.e., a grade that y actually received. Setting $g_8 \equiv [\![\text{a}]\!]^g$, as it is eventually resolved to once the focus binds its trace, would render the denotation of the superlative's complement:

$$[\![\text{a [5 [[}\textit{the}\text{ grade] [}\star_2\text{ [}\eta\text{ [}t_5\text{ received }t_2\text{]]]]]}]\!]^g = \left\{\left\langle y,\ g^{2\mapsto x,\ 5\mapsto y}\right\rangle \;\middle|\; \text{grd}(x) \wedge \text{rcv}(x)(y)\right\}$$

At this point the only entities left among the outputs as potential values for g_2 are the grades *that were received* by some alternative to John. The superlative $\textit{highest}_2$ then filters out all but those outputs that are maximal in their choice of 2. This leaves only those assignments $g^{2\mapsto x,\ 5\mapsto y}$ such that x received y and no other grade that was received (by anyone) was higher than y. Each such assignment is paired with the entity y that received it. Given the assumption we have made that the superlative is only felicitous if all remaining outputs after its execution have the same value component — only if there is exactly one "winner" — we predict that this highest received grade must have been received by exactly one person.

Finally, the focus semantics completes the derivation by checking to see whether John is among those that received this highest grade. If so, the sentence returns true, together with an updated assignment listing John and John's highest grade as the witnessing values for the focus and the superlative description, respectively.

(60) $[\![\text{JOHN received the highest grade}]\!]^g$



Before we wrap up, a reviewer points out that the story above is likely to encounter difficulty with Heim's (1999) split-scope *de dicto* cases, as in (61).

(61) John needs to climb the highest mountain

a. ✓John has a mountain-climbing height requirement that exceeds everyone else's mountain-climbing height requirement

The reading of (61) paraphrased in (61a) is generally challenging for any theory of superlatives that achieves relative readings without scoping the superlative morpheme out of its description. The chief issue is that the predicative components of the description can be interpreted in the scope of the attitude, while the superlative comparison is nevertheless between what different people's needs are. Bumford's (2017a) analysis, even though it relies on superlative scope-taking, appears to be similarly at a loss here, since the superlative scopes together with the lexical adjective. It is not obvious how the height of a mountain, which can vary form world to world, should be determined if the adjective is evaluated above the attitude. In fact, there seem to be deeper questions

about how the binding information in the embedded clause could possibly leak out into the matrix, given that attitudes are canonically envisioned as externally static. We take the reviewer's point here, and admit that Heim (1999) remains in the best position to account for the split-scope truth conditions in (61) even while it is unable to account for the distribution of NPIs we have explored.

7 Conclusion

In this article, we have pointed out a basic tension between theories of superlative ambiguity on the one hand and theories of superlative NPI-licensing on the other, largely following the lead of Howard (2014). Accounts of the latter have concentrated on absolute readings, and have tended to analyze almost all of the superlative's noun phrase as contributing to its semantic restrictor, which typically serves as the comparison class of objects that the superlative measures. Accounts of the former capture both readings, but have tended to analyze almost all of superlative's noun phrase, or even the entire clause containing the superlative's noun phrase, as contributing to its semantic scope, which typically serves as the degree-entity relation that the superlative uses to measure its comparison class. The restrictor argument is approximately downward entailing, but the scope argument is not. This leaves many successful theories of the scope ambiguity unable to explain the grammaticality of NPIs in superlative descriptions, and many successful theories of NPI-licensing unable to derive relative readings.

Howard (2014) found a way to make progress on the NPI front without giving up on a scopally-mobile superlative operator. His insight was that in some sentences, a superlative's comparison class may be spelled out by an entire alternative-generating clause. For instance, in (62a) the relative clause inside the superlative noun phrase is taken to denote the set of degree properties, one per person here, mapping each entity to the lengths of books they read. The sentence is true if the degree property identifying the lengths of books Mary read is the longest of these. This predicts that NPI-licensing in relative superlative descriptions depends on the NPI-containing constituent serving up a set of degree properties that includes the degree property formed by abstracting over the superlative in the matrix. This explains why (62b) sounds like nonsense: if the comparison class C only contains degree properties corresponding to people other than Mary, then Mary's own degree property can't possibly be the largest in C, because it isn't in there at all!

(62) a. Out of all of us, MARY has read the longest paper that anyone here has read
b. *Out of all of us, MARY has read the longest paper that anyone else read

However, we argued that this analysis comes at the cost of having to posit different superlative morphemes for absolute and relative readings. And it does not account for legitimate NPIs that appear outside the superlative's own noun phrase, i.e., those that fall under our Generalization II, including (63a,b).

(63) a. Out of all of us, MARY has written the longest paper while advising anyone's dissertation
b. Out of all of us, MARY has written the longest paper containing any claim from her dissertation

After failing to find a charitable extension of Howard's (2014) technique, we considered a dynamic account of superlatives due to Bumford 2017a. This analysis has two desirable features from the perspective of the puzzles here: (i) as in Howard 2014, following Heim 1999, superlatives take genuine scope over the constituents that influence their meaning; and (ii) as in von Fintel 1999, the superlative combines with its gradable measure first, independently of anything else in the sentence. The first feature ensures that superlatives will have a chance to license description-external NPIs. The second ensures that everything that the superlative adjective scopes over is in a properly SDE argument.

But what of Howard's "matching" judgments? Well, we certainly cannot maintain the generalization that an NPI in a relative superlative noun phrase is necessarily ungrammatical when its own argument position differs from that of the superlative's focus. After all, (63b) and other examples in Section 4.2 plainly contradict this.

Yet, we agree that Howard's non-matching examples can be understood in ways that make them sound deviant. That is, there is a way of hearing (62b) such that it presupposes — nonsensically — that Mary is among the set of people who are not Mary, and of these people who are not her, it says that Mary has read the longest paper. (We speculate that it is most natural with stress on *anyone*.) But strictly speaking, we think that the example is at least ambiguous. It can also be understood — sensically — as saying that if you look at just those papers that have been read by one of us z and also another person $y \neq z$, Mary has read the longest. (We speculate that this reading is most natural with stress on *else*.)[18]

In any case, if it's true that an NPI-clause can be interpreted as delivering an explicit comparison class for the superlative, then perhaps we should seek to refashion Howard's derivations so that the relevant clauses denote sets of alternative *updates*, rather than alternative degree properties. We see no inherent obstacle to implementing this idea with Bumford's (2017a) target types in mind, but we leave the exercise for future work.

References

Barker, Chris. 2007. Parasitic scope. *Linguistics and Philosophy* 30(4). 407–444. https://doi.org/10.1007/s10988-007-9021-y.

Bobaljik, Jonathan David. 2012. *Universals in comparative morphology: Suppletion, superlatives, and the structure of words*. Vol. 50 (Current Studies in Linguistics). Cambridge, MA: MIT Press. https://doi.org/10.7551/mitpress/9069.001.0001.

Brasoveanu, Adrian. 2013. Modified numerals as post-suppositions. *Journal of Semantics* 30(2). 155–209. https://doi.org/10.1093/jos/ffs003.

Bumford, Dylan. 2017a. Split-scope definites: Relative superlatives and Haddock descriptions. *Linguistics and Philosophy* 40(6). 549–593. https://doi.org/10.1007/s10988-017-9210-2.

Bumford, Dylan. 2017b. *Split-scope effects in definite descriptions*. New York, NY: New York University PhD Dissertation.

Bumford, Dylan. 2018. Binding into superlative descriptions. In *Semantics and linguistic theory*, vol. 28, 325–344. https://doi.org/10.3765/salt.v28i0.4412.

Charlow, Simon. 2019. Where is the destructive update problem? To appear at *Semantics & Pragmatics*.

Chierchia, Gennaro. 2013. *Logic in grammar: Polarity, free choice, and intervention*. Oxford University Press. https://doi.org/10.1093/acprof:oso/9780199697977.001.0001.

Coppock, Elizabeth & David Beaver. 2014. A superlative argument for a minimal theory of definiteness. In *Semantics and linguistic theory (SALT) 24*, 177–196. https://doi.org/10.3765/salt.v24i0.2432.

Cresswell, Maxwell J. 1976. The semantics of degree. In Barbara Partee (ed.), *Montague Grammar*, 261–92. New York, NY: Academic Press. https://doi.org/10.1016/B978-0-12-545850-4.50015-7.

Crnič, Luka. 2014. Non-monotonicity in NPI licensing. *Natural Language Semantics* 22(2). 169–217. https://doi.org/10.1007/s11050-014-9104-6.

Farkas, Donka & Katalin E. Kiss. 2000. On the comparative and absolute readings of superlatives. *Natural Language and Linguistic Theory* 18(3). 417–455. https://doi.org/10.1023/A:1006431429816.

von Fintel, Kai. 1997. Bare plurals, bare conditionals, and *only*. *Journal of Semantics* 14(1). 1–56. https://doi.org/10.1093/jos/14.1.1.

von Fintel, Kai. 1999. NPI licensing, Strawson Entailment, and context dependency. *Journal of Semantics* 16. 97–148. https://doi.org/10.1093/jos/16.2.97.

Fitzgibbons, Natalia, Yael Sharvit & Jon Gajewski. 2008. Plural superlatives and distributivity. In Tova Friedman & Satoshi Ito (eds.), *Semantics and linguistic theory (SALT) 18*, 284–301. The University of Massachusetts, Amherst. https://doi.org/10.3765/salt.v18i0.2501.

18 Presumably the sentence actually has two relative readings, depending on whether the *else* is interpreted strictly or sloppily (see Bumford 2018). The latter is given by the paraphrase in the main text. The former would say that of the papers that have been read by one of us and also somebody who's not Mary, Mary has read the longest.

Fox, Danny. 2003. On Logical Form. In Randall Hendrick (ed.), *Minimalist syntax* (Generative Syntax 7). Malden, MA: Blackwell Publishers. https://doi.org/10.1002/9780470758342.ch2.
Gajewski, Jon. 2005. *Neg-raising: Polarity and presupposition*. Cambridge, MA: Massachusetts Institute of Technology PhD Dissertation.
Gajewski, Jon. 2007. Neg-raising and polarity. *Linguistics and Philosophy* 30(3). 289–328. https://doi.org/10.1007/s10988-007-9020-z.
Gajewski, Jon. 2010. Superlatives, NPIs and *most*. *Journal of Semantics* 27(1). 125–137.
Gajewski, Jon. 2011. Licensing strong NPIs. *Natural Language Semantics* 19(2). 109–148. https://doi.org/10.1007/s11050-010-9067-1.
Gajewski, Jon & I-ta Chris Hsieh. 2014. Comments on Negative Polarity Items in definite description. In Luka Crnic & Uli Sauerland (eds.), *The art and craft of semantics: A festschrift for Irene Heim*, 181–198.
Guerzoni, Elena & Yael Sharvit. 2007. A question of strength: On NPIs in interrogative clauses. *Linguistics and Philosophy* 30(3). 361–391. https://doi.org/10.1007/s11050-010-9067-1.
Heim, Irene. 1984. A note on negative polarity and downward entailingness. In *North east linguistic society (NELS) 14*, 98–107.
Heim, Irene. 1985. Notes on comparatives and related matters. Ms., University of Texas at Austin. https://semanticsarchive.net/Archive/zc0ZjY0M/Comparatives%2085.pdf.
Heim, Irene. 1997. Predicates or formulas? evidence from ellipsis. In *Semantics and linguistic theory (SALT)*, vol. 7, 197–221. https://doi.org/10.3765/salt.v7i0.2793.
Heim, Irene. 1999. Notes on superlatives. Ms., MIT. https://semanticsarchive.net/Archive/TI1MTlhZ/Superlative.pdf.
Heim, Irene. 2018. Functional readings without type-shifted noun phrases. *Reconstruction Effects in Relative Clauses* 75. 283. https://doi.org/10.1515/9783050095158-009.
Heim, Irene & Angelika Kratzer. 1998. *Semantics in Generative Grammar*. Oxford: Blackwell.
Homer, Vincent. 2008. Presuppositions can be disruptors too: A case against Strawson-entailment. In *West coast conference on formal linguistics (WCCFL) 27*, vol. 27, 220–228.
Howard, Edwin. 2014. *Superlative degree clauses: Evidence from NPI licensing*. Cambridge, MA: Massachusetts Institute of Technology MA thesis.
Karttunen, Lauri. 1977. Syntax and semantics of questions. *Linguistics and Philosophy* 1(1). 3–44. https://doi.org/10.1007/BF00351935.
Kennedy, Christopher & Jason Stanley. 2009. On 'average'. *Mind* 118(471). 583–646. https://doi.org/10.1093/mind/fzp094.
Kratzer, Angelika & Junko Shimoyama. 2002. Indeterminate pronouns: The view from Japanese. In Yukio Otsu (ed.), *Third Tokyo conference on psycholinguistics*, 1–25. Tokyo. https://doi.org/10.1007/978-3-319-10106-4_7.
Krifka, Manfred. 1995. The semantics and pragmatics of polarity items. *Linguistic Analysis* 25(3-4). 209–257.
Ladusaw, William. 1979. *Polarity sensitivity as inherent scope relations*. Austin, TX: University of Texas at Austin PhD Dissertation.
Lahiri, Utpal. 1998. Focus and negative polarity in Hindi. *Natural Language Semantics* 6(1). 57–123. https://doi.org/10.1023/A:1008211808250.
Loccioni, Nicoletta. 2018. *Getting "the most" out of Romance*. Los Angeles, CA: University of California, Los Angeles dissertation.
Morzycki, Marcin. 2016. Toward a general theory of nonlocal readings of adjectives. In *Sinn und bedeutung (SuB) 21*, vol. 20, 515–532.
Partee, Barbara. 1986. Noun phrase interpretation and type-shifting principles. In Jeroen Groenendijk, Dick de Jongh & Martin Stokhof (eds.), *Studies in Discourse Representation Theory and the theory of generalized quantifiers*, 115–144. Dordrecht: Foris. https://doi.org/10.1002/9780470751305.ch10.

Romero, Maribel. 2013. Modal superlatives: A compositional analysis. *Natural Language Semantics* 21(1). 79–110. https://doi.org/10.1007/s11050-012-9090-5.
Rooth, Mats. 1992. Ellipsis redundancy and reduction redundancy. In Steve Berman & Arild Hestvik (eds.), *Stuttgart ellipsis workshop*.
Ross, John Robert. 1964. *A partial grammar of English superlatives*. Philadelphia, PA: University of Pennsylvania MA thesis.
Rullmann, Hotze. 1995. *Maximality in the semantics of wh-constructions*. Amherst, MA: University of Massachusetts at Amherst PhD Dissertation.
Sharvit, Yael & Penka Stateva. 2002. Superlative expressions, context, and focus. *Linguistics and Philosophy* 25(4). 453–505. https://doi.org/10.1023/A:1020875809794.
Sharvy, Richard. 1980. A more general theory of definite descriptions. *The Philosophical Review* 89(4). 607–624. https://doi.org/10.2307/2184738.
Stateva, Penka. 2002. *How different are different degree constructions?* University of Connecticut PhD Dissertation.
von Stechow, Arnim. 1984. Comparing semantic theories of comparison. *Journal of Semantics* 3(1-2). 1–77. https://doi.org/10.1093/jos/3.1-2.1.
Szabolcsi, Anna. 1986. Comparative superlatives. In *MIT Working Papers in Linguistics 8*, 245–265.
Wilson, E. Cameron. 2018. *Amount superlatives and measure phrases*. New York, NY: City University of New York dissertation.
Zimmermann, Malte. 2003. Pluractionality and complex quantifier formation. *Natural Language Semantics* 11(3). 249–287. https://doi.org/10.1023/A:1024937316555.

A Appendix: Derivations with *ever*

For completeness, we provide derivations involving the NPI *ever* analogous to those involving *any* in the main text. We do this for this two reasons. First, *ever* is an unambiguous NPI, unlike *any*, so it is important in establishing that our patterns of interest are driven by polarity rather than Free Choice. Second, Howard (2014) makes extensive use of *ever* examples in his arguments, but does not provide any explicit analyses of these, so we seek to fill this gap with concrete LFs.

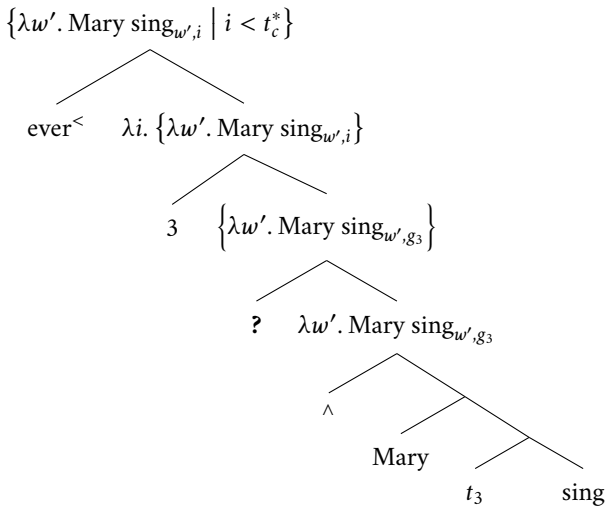
Not wishing to wade very deep into the semantics of tense, we assume that when *ever* modifies a simple past-tense predicate, it binds the tense argument of that predicate, quantifying over a property of intervals. Using this property, it generates a set of alternatives, one for each (relevant) interval in the past of the contextual utterance time. Because the utterance time is indexical, we add a context parameter c to denotation function $[\![\cdot]\!]^{c,w,g}$, and represent the utterance time relative to this context as t^*_c. We do not deal with *ever* in perfect or non-past phrases, though we would assume that this alters which time intervals are used to generate alternatives. The metalanguage expression "$\text{sing}_{w,i}$" is intended to describe singing events at time i in world w.

Without *ever*, the simple past is assumed to introduce a free pronominal interval argument to its predicate. This interval is presupposed to precede the utterance time.

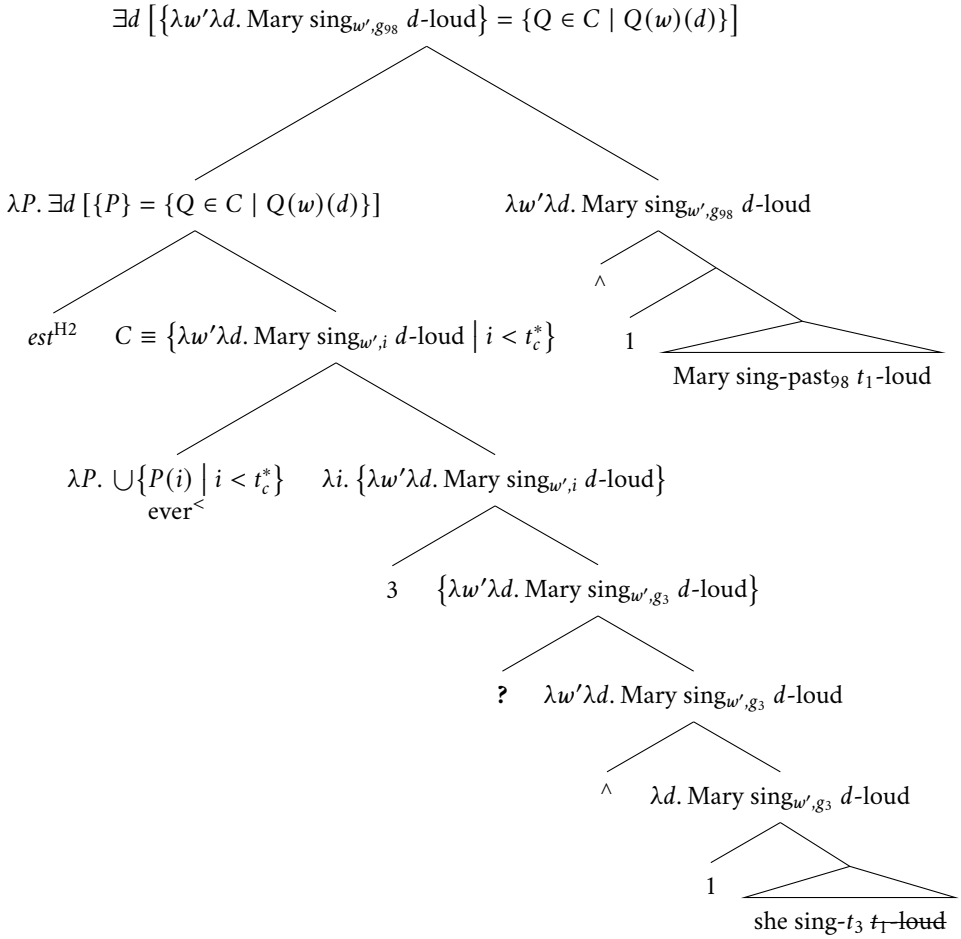
(64) a. $[\![\text{ever}^{<}]\!]^{c,w,g} := \lambda P.\ \bigcup\{P(i) \mid i < t^*_c\}$
b. $[\![\text{past}_n]\!]^{c,w,g}$ is defined only if $g_n < t^*_c$. If defined, $[\![\text{past}_n]\!]^{c,w,g} := g_n$

An example derivation of a clause containing *ever* is given in (65). This construction is put to use in the derivation of a Howard-style superlative ACD clause in (66).

(65) $[\![\text{Mary ever sang}]\!]^{c,w,g}$



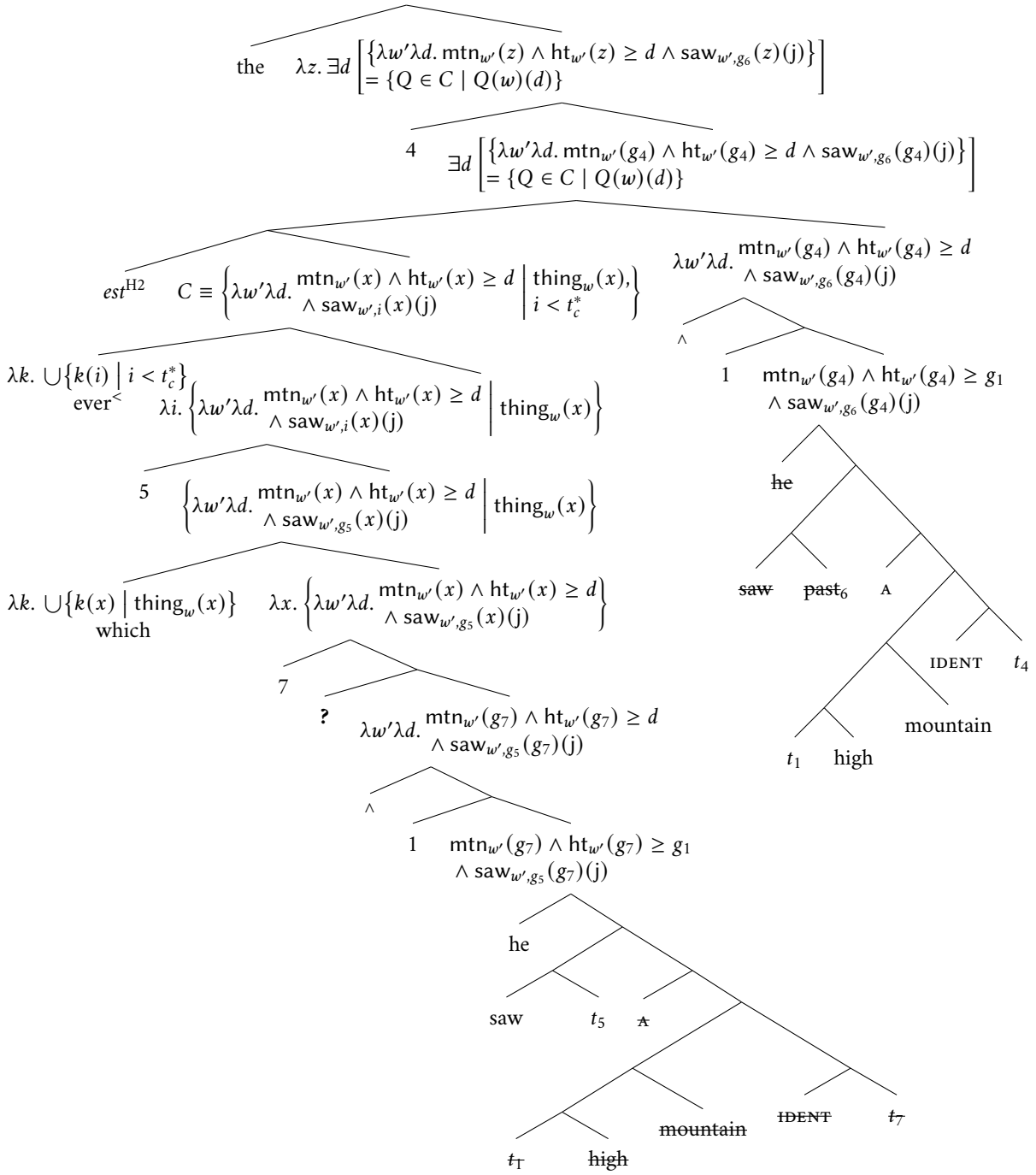
(66) (In 1998,) $[\![\text{Mary sang the loudest she ever sang}]\!]^{c,w,g}$



Like the example with *any* discussed in the main text (34), the closest structure we can find for an ACD-like derivation of an absolute description including *ever* is not empirically adequate. The truth conditions of the LF

in (67) are too strong. They require that John only saw the referent of the superlative description once. If he saw every mountain on two different occasions, for instance, then the sentence will be predicted to have an unsatisfiable presupposition. But this is a bad prediction. As long as one of the mountains John saw is higher than all the others he saw, the sentence is well-defined, regardless of how many times John saw the mountain in question.

(67) $\exists_6$ [John climbed $[\![$the highest mountain he ever saw$]\!]^{c,w,g}$]



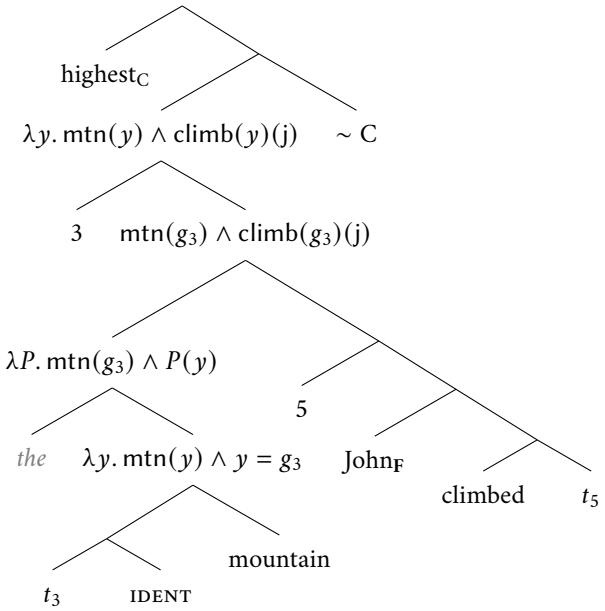
B Appendix: Scoping the adjective with the superlative is not enough

Here we show that the dynamic fragment introduced in Section 6 to define a superlative with an SDE scope is at least not easily approximated by a static lexical entry. Let *highest* be defined as in (68).

(68) $[\![\text{highest}]\!]^g := \lambda C \lambda P\colon P \in C \wedge \forall Q \in C\,[\exists y\,[P(y)]]$
$.\,\exists y\,[P(y) \wedge \exists d\,[\{y\} = \bigcup\{\{y' \mid Q(y') \wedge \text{ht}(y') \geq d\} \mid Q \in C\}]]$

This would lead to LFs like the following, where the superlative's restrictor C is constrained by the sentence's focus alternatives to be a set of properties of the form $\lambda y'.\,\text{mtn}(y') \wedge \text{climb}(y')(z)$, for some $z \in \mathcal{D}_e$. The entire sentence is predicted to presuppose that John climbed a mountain, and to be true iff John climbed a mountain higher than any other mountain climbed by anyone.

(69) $[\![\text{JOHN climbed the highest mountain}]\!]^g$
$\exists y\,[\text{mtn}(y) \wedge \text{climb}(y)(\text{j}) \wedge \exists d\,[\{y\} = \{y' \mid \exists z\,[\text{mtn}(y') \wedge \text{climb}(y')(z)]\}]]$



The problematic NPIs of Generalization II would appear in P, the nuclear scope argument of this superlative (since the comparison class is given implicitly by association with focus), so we need to show that this argument is not SDE. Consider the model below, with five climbers, five mountains, and two properties P_z and Q_z per climber.

$P_z := \lambda y.\ y$ is a mountain that z climbed yesterday
$Q_z := \lambda y.\ y$ is a mountain that z climbed this week

z	$\{\text{ht}(y) \mid P_z(y)\}$	$\{\text{ht}(y) \mid Q_z(y)\}$
ann	$\{20\}$	$\{25, 20\}$
bud	$\{20\}$	$\{30, 20\}$
cal	$\{35\}$	$\{60, 35\}$
deb	$\{50\}$	$\{55, 50\}$
eve	$\{30\}$	$\{35, 30\}$

In this world, everybody climbed a mountain yesterday, but everybody climbed an even higher mountain two days ago in addition to the one they also climbed yesterday. Let C contain for each climber z both the property of mountains that z climbed yesterday and the property of mountains that z climbed this week:

$$C := \bigcup \left\{ \{P_z, Q_z\} \;\middle|\; z \in \{\text{ann}, \ldots, \text{eve}\} \right\}$$

Let $P \equiv P_{\text{cal}}$ and $Q \equiv Q_{\text{cal}}$. Clearly P and Q are both in C, and none of the properties in C are empty. So both $[\![\text{highest}]\!]^g(C)(P)$ and $[\![\text{highest}]\!]^g(C)(Q)$ are defined. And clearly $P \Rightarrow Q$, since the set of mountains Cal climbed yesterday is a subset of the set of mountains he climbed this week. Moreover, $[\![\text{highest}]\!]^g(C)(Q)$ is true, since Cal climbed a mountain this week higher than any other mountains that were climbed this week (namely, the one of size 60 that he climbed two days ago). But $[\![\text{highest}]\!]^g(C)(P)$ is false; Cal didn't climb a mountain yesterday higher than any other mountain climbed yesterday (he was beat by Deb). So $[\![\text{highest}]\!]^g(C)(Q) \overset{\text{ST}}{\not\Rightarrow} [\![\text{highest}]\!]^g(C)(P)$, and thus $[\![\text{highest}]\!]^g(C)$ is not SDE.

To be clear, association with focus is not the culprit here. Consider an *est*$^{\text{H1}}$-inspired superlative (70) in essentially the same configuration (71).

(70) $[\![\text{highest}]\!]^g := \lambda R \lambda x\colon \exists y\,[R(y)(x)]$
$.\,\exists y\,[R(y)(x) \wedge \exists d\,[\{y\} = \{y' \mid \exists x'\,[R(y')(x') \wedge \mathsf{ht}(y') \geq d]\}]]$

The same model as above, together with the relations R and S, suffice to show that this version of $[\![\text{highest}]\!]^g$ is not SDE on its scope either.

$$R := \lambda y \lambda x.\ y \text{ is a mountain that } x \text{ climbed yesterday}$$
$$S := \lambda y \lambda x.\ y \text{ is a mountain that } x \text{ climbed this week}$$

If x climbed y yesterday, then x climbed y this week, so $R \Rightarrow S$. But even if we assume that x climbed a mountain yesterday, it doesn't follow from the fact that x climbed a mountain this week higher than any other mountain climbed this week that the mountain x climbed *yesterday* was higher than any other mountain climbed *yesterday*.

Note that if we also knew in advance that everybody climbed exactly one mountain this week, so that S^{-1} was in fact a function, and R^{-1} a partial function, then $[\![\text{highest}]\!]^g$ *would* be SDE (but not SUE) on its scope. In this case, knowing that x climbed the highest of the mountains climbed this week, and x climbed *that mountain* (rather than merely *some mountain*) yesterday, does guarantee that x climbed the highest of the mountains climbed yesterday. But in general, relative readings are quite natural in contexts where the association relation is non-functional. And more to the point, the grammaticality of NPIs is not conditioned by whether or not this relation is functional. For instance, *JOHN gave the highest score to any rookie* is still grammatical when it is known that every judge assigned every player a different score (or even multiple different scores), so that the relation between judges and scores they assigned is one-to-many.

(71) $[\![\text{JOHN climbed the highest mountain}]\!]^g$

$$\exists y \left[\begin{array}{l}\mathsf{mtn}(y) \wedge \mathsf{climb}(y)(\mathrm{j}) \wedge \\ \exists d\,[\{y\} = \{y' \mid \exists x'\,[\mathsf{mtn}(y') \wedge \mathsf{climb}(y')(x') \wedge \mathsf{ht}(y') \geq d]\}]\end{array}\right]$$