Split-scope definites

How ‘the’ can mean two things at once

Dylan Bumford
18 February 2016

New York University
**Wisdom:** ‘the NP’ refers to the single salient ‘NP’ in the context 
\[\text{[the hat]} = x, \text{where } x \text{ is the unique relevant hat}\]

**Proposal:** Definite determination split into two subprocesses. 
\[\text{[the hat]} = \text{one} (\cdots (\text{some} \text{ hat}))\]

When things intervene, ‘the hat’ may end up one among many

**Payoffs:**

- Haddock readings
- Relative superlatives
- Possibly other strange readings of quantificational adjectives
- Emerging uniformity in the theory of cardinal modification
Definite description

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Haddock descriptions

(1) the rabbit in the hat  

![Image of rabbit in hat](image)

What about H2?

cf. #The hat is my favorite

(2) the table with the apple and the banana  

![Image of table with fruits](image)

Nothing especially salient about the relevant fruit
(1) the rabbit in the hat

[Horacek 1995]

the table with the apple and the banana

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Constraint Satisfaction Problem

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Unique x and y satisfying these simultaneous constraints
Noncompositional. Worse, circular!
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Unique x and y satisfying these simultaneous constraints

Noncompositional. Worse, circular!
Relative superlatives

(2)  the girl who got the fewest letters  [Szabolcsi 1986]

(3)  a.  *When was there the rabbit in the garden?
   b.  When were there the most rabbits in the garden?

(4)  the rabbit in the biggest hat
(2) the girl who got the fewest letters

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(5) the rabbit in the biggest hat
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Decomposing definiteness

The basic idea: definiteness is a two-step process

\[
G \quad \text{if } \left\{ g \nu \mid g \in G \right\} = 1 \\
\# \quad \text{otherwise}
\]
The basic idea: definiteness is a two-step process

\[
\begin{cases}
G & \text{if } \left| \{g \nu \mid g \in G\} \right| = 1 \\
\# & \text{otherwise}
\end{cases}
\]

\[\begin{array}{c}
\text{the}_\nu \\
\text{some}_\nu \\
\end{array}
\begin{array}{c}
\{ \nu \mapsto x \mid \text{hat } x \} \\
\text{hat}
\end{array}\]
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Dynamic Semantics

- Denotations are sets of assignments
- Indefinites introduce *nondeterministic* referents
Decomposing definiteness

The basic idea: definiteness is a two-step process

\[ G \text{ if } \left| \{ g \nu \mid g \in G \} \right| = 1 \]
\[ # \text{ otherwise} \]

```
\nu \mapsto x \mid \text{hat } x
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\[\nu \mapsto x \mid \hat{x}\]

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\text{some}_\nu
\end{array}
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\[
\text{hat}
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- Denotations are sets of assignments
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The basic idea: definiteness is a two-step process

\[
\begin{cases}
G & \text{if } \left| \{ g, \nu \mid g \in G \} \right| = 1 \\
\# & \text{otherwise}
\end{cases}
\]
Teasing the pieces apart

(6) the [rabbit in the hat]

(7) the [rabbit in the biggest hat]
Teasing the pieces apart

(6) the [rabbit in the hat]

(7) the [rabbit in the biggest hat]
Teasing the pieces apart

(6) \textcolor{red}{\textbf{one}} \ [\textcolor{blue}{\textbf{some}} \ \text{rabbit in} \ [\textcolor{red}{\textbf{one}} \ [\textcolor{blue}{\textbf{some}} \ \text{hat}]]]

(7) \textcolor{blue}{\textbf{the}} \ [\text{rabbit in the biggest hat}]
Teasing the pieces apart

some some

(6) the [rabbit in the hat]

one one

(7) the [rabbit in the biggest hat]
Teasing the pieces apart

(6) one [one [ some rabbit in some hat]]

(7) the [rabbit in the biggest hat]
Teasing the pieces apart

(6)  one [one [ some rabbit in some hat]]

(7)  the [rabbit in the biggest hat]
Teasing the pieces apart

(6) the [rabbit in the hat]

(7) the [rabbit in the biggest hat]
Teasing the pieces apart

(6) the [rabbit in the hat]

some

some

(7) the [rabbit in the biggest hat]

one

one biggest
Teasing the pieces apart

(6) the [rabbit in the hat]

(7) one [some rabbit in [one biggest [some hat]]]
Teasing the pieces apart

(6) the [rabbit in the hat]

some some

(7) the [rabbit in the biggest hat]

one one biggest
(6) the [rabbit in the hat]

(7) one [one biggest [ some rabbit in some hat]]
(6)  the [rabbit in the hat]

(7)  one [one biggest [ some rabbit in some hat]]
Haddock effects: Interleaved definites

\[ \begin{align*}
\text{the}_u & \quad 1_u \\
\text{1}_v & \quad \{ v \mapsto x \mid \text{hat } x, \text{ rab } y, \text{ in } x \, y \} \\
\text{some}_u & \quad \text{rabbit} \\
\text{the}_v & \quad \text{in} \quad \{ v \mapsto x \mid \text{hat } x \} \\
& \quad \text{some}_v \\
& \quad \text{hat}
\end{align*} \]
Haddock effects: Interleaved definites

1. Haddock effects involve the interaction of definites within a sentence structure.

2. The diagram illustrates the sentence structure with nodes labeled by definites and quantifiers.

3. Each node represents a specific definiteness type:
   - \(1_u\) for universal
   - \(1_v\) for existential
   - \(\text{the}_u\)
   - \(\text{the}_v\)
   - \(\text{some}_u\)
   - \(\text{some}_v\)

4. The relationships between definites and quantifiers are shown through mappings:
   - \(\nu \mapsto x\) indicating the definiteness of a term.

5. The diagram includes an example of how definites can be interleaved within a sentence, indicating how definites interact with each other.

6. The sentence structure is further explained with examples and mappings, showing how definites are assigned to specific positions in the sentence.

7. The diagram also includes a visual representation of the sentence structure, with nodes and edges indicating the relationships between definites and quantifiers.
Haddock effects: Interleaved definites
Haddock effects: Interleaved definites
Haddock effects: Interleaved definites

\[
\begin{align*}
1_u & \quad \left\{ v \mapsto x \mid \text{hat } x, \ \text{rab } y, \ \text{in } x \ y \right\} \\
\text{the}_u & \\
1_v & \quad \left\{ u \mapsto y \right\} \\
\text{the}_v & \\
\text{some}_u & \quad \text{rabbit} \\
\text{in} & \quad \left\{ v \mapsto x \mid \text{hat } x \right\} \\
\text{some}_v & \quad \text{hat}
\end{align*}
\]
Haddock effects: Interleaved definites

\[ \{ v \mapsto x \mid \text{hat } x, \ rab \ y, \ \text{in } x \ y \} \]

\[ \{ v \mapsto x \mid \text{hat } x \} \]

\[ \{ v \mapsto x \mid \text{hat } x \} \]
Haddock effects: Interleaved definites

\[ \{ v \mapsto x \mid \text{hat } x, \text{ rab y, in } x \text{ y} \} \]

\[ \{ v \mapsto x \mid \text{hat } x \} \]

\[ \text{rabbit} \]

\[ \text{in} \]

\[ \{ v \mapsto x \mid \text{hat } x \} \]

\[ \text{some}_u \]

\[ \text{the}_u \]

\[ 1_u \]

\[ \text{some}_v \]

\[ \text{hat} \]

\[ \text{the}_v \]
Haddock effects: Interleaved definites

\[
\begin{align*}
1_u & \quad \{
\nu \mapsto x \quad \text{hat } x, \quad \text{rab } y, \quad \text{in } x \ y
\} \\
\text{the}_u & \\
1_v & \\
\text{the}_v & \\
\text{some}_u & \\
\text{rabbit} & \\
\text{in} & \quad \{
\nu \mapsto x \quad \text{hat } x
\} \\
\text{some}_v & \\
\text{hat} &
\end{align*}
\]
Relative superlatives: Delayed maximality filter

\[ 1_u \circ S_v \{ v \mapsto x | \text{hat} x, \text{rab} y, \text{in} x y \} \]

\[ \text{the}_u \]

\[ \text{some}_u \]

\[ \text{the biggest}_v \]

\[ \text{rabbit} \]

\[ \text{in} \{ v \mapsto x | \text{hat} x \} \]

\[ \text{some}_v \]

\[ \text{hat} \]
Relative superlatives: Delayed maximality filter

\[ 1_u \circ S_v \{ \nu \mapsto x \mid \text{hat } x, \text{ rab } y, \text{ in } x y \} \]

- **the** \( u \)
- **1** \( u \)
- **some** \( u \)
Relative superlatives: Delayed maximality filter

\[ 1_u \circ S_v \{ v \mapsto x \mid \text{hat } x, \text{rab } y, \text{in } x y \} \]

\[ \text{the}_u \]

\[ \text{the biggest}_v \]

\[ \text{some}_u \]

\[ \text{rabbit} \]

\[ \text{in} \{ v \mapsto x \mid \text{hat } x \} \]

\[ \text{some}_v \]

\[ \text{hat} \]
Relative superlatives: Delayed maximality filter

\[ 1_u \circ S_v \{ v \mapsto x \mid \text{hat } x, \text{ rab } y, \text{ in } x y \} \]

- the\( u \)
- \( 1_v \circ S_v \)
- some\( u \)
- the biggest\( v \)
- rabbit
- in \( \{ v \mapsto x \mid \text{hat } x \} \)
- some\( v \)
- hat
Relative superlatives: Delayed maximality filter

\[ \text{the}_u \quad 1_u \quad \text{the}_v \circ S_v \quad \{ v \mapsto x \mid \text{hat } x, \text{ rab } y, \text{ in } x y \} \]

\[ \text{some}_u \]

\[ \text{the biggest}_v \]

\[ \text{rabbit} \quad \text{in} \quad \{ v \mapsto x \mid \text{hat } x \} \]

\[ \text{some}_v \quad \text{hat} \]
Relative superlatives: Delayed maximality filter

\[ 1_u \circ S_v \{ \nu \mapsto x \mid \text{hat } x, \text{ rab } y, \text{ in } x y \} \]

Some \( u \) rabbit in \( \{ \nu \mapsto x \mid \text{hat } x \} \)

The biggest \( v \) in \( \{ \nu \mapsto x \mid \text{hat } x \} \)

The \( u \)
Relative superlatives: Delayed maximality filter

\[ 1_u \circ S_v \{ \nu \mapsto x \mid \text{hat } x, \text{ rab } y, \text{ in } x y \} \]

the \[ v \]

the biggest \[ v \]

some \[ u \]

rabbit

in \{ \nu \mapsto x \mid \text{hat } x \}

some \[ v \]

hat
Relative superlatives: Delayed maximality filter

Superlative as filter

Keep only the assignments that are undominated in their choice of $\nu$

$$S_\nu G := \{ g \in G \mid \neg \exists g' \in G. \ g' \nu > g \nu \}$$
Relative superlatives: Delayed maximality filter

$1_u \circ S_v \{ v \mapsto x | \text{hat } x, \text{rab } y, \text{in } x \ y \}$

the $u$

the biggest $v$

some $u$

rabbit

in $\{ v \mapsto x | \text{hat } x \}$

some $v$

hat
Relative superlatives: Delayed maximality filter

\[ 1_u \circ S_v \{ v \mapsto x \mid \text{hat } x, \text{ rab } y, \text{ in } x y \} \]

the \( u \)

\[ 1_v \circ S_v \{ v \mapsto x \mid \text{hat } x, \text{ rab } y, \text{ in } x y \} \]

\[ \text{the} \ u \]

\[ \text{the biggest}_v \]

\[ \text{some}_u \]

\[ \text{rabbit} \]

\[ \text{in} \ \{ v \mapsto x \mid \text{hat } x \} \]

\[ \text{some}_v \]

\[ \text{hat} \]
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\[ \text{some}_u \]

\[ \text{the biggest}_v \]

\[ \text{rabit} \]

\[ \text{in} \{ v \mapsto x \mid \text{hat } x \} \]

\[ \text{some}_v \]

\[ \text{hat} \]
Connections and applications: Quantificational adjectives

Range of quantificational adjectives that ride on the scope of the definite article

(8) John gave Mary the first telescope  [Bylinina et al. 2014]
   a. John was the first to give Mary a telescope

(9) Mary didn’t score the only goal  [Coppock & Beaver 2015]
   a. Mary wasn’t the only one to score a goal

(10) Ann read the same book yesterday and today  [Barker 2007]
   a. Ann read a book yesterday and a book today; they were the same
And more generally, cardinality-testing denotations appear happy to take delayed action

(11) You should talk to at least three professors [Cresti 1995]
    a. You should talk to some professors; three at the least

(12) Exactly three boys saw exactly five movies [Brasoveanu 2012]
    a. Some boys saw some movies; three and five, to be exact
Plenty of constructions known to contribute two kinds of meaning at once

- Focus
  
  *I gave the book to JOHN*

- Conventional Implicature and presupposition
  
  *John, a linguist, received a mysterious book*

- Anaphora and discourse referent management
  
  *A man walked in; he asked John about his book*

- Alternative generation
  
  *John either liked or hated his book; I can’t remember*
Scope as multidimensional meaning

- Quantification is a kind of multidimensional effect
  
  \( \text{every}_x \text{ student} \)
  
  John talked to \( x \)

- Definiteness is just like that, but more
  
  \( \text{one}_u \)

  \( \text{some}_u \)
  
  John talked to \( u \)
Conclusion

- Definiteness is semantically bipartite

\[
\text{the}_v \quad 1_v \quad \text{some}_v \quad \text{hat}
\]

- Mismatches in the execution of the parts accounts for relative readings of definites and superlatives, and possibly other quantificational adjectives

- Encourages a multidimensional view of meaning, in which different subprocesses of a denotation may act at different times on different arguments
Thanks!

Thanks!


\[
\begin{align*}
m \parallel n & := \begin{cases} 
m n & \text{if } m :: \alpha \to \beta, \ n :: \alpha \\
\lambda k. m (\lambda f. n (\lambda x. k (f \parallel x))) & \text{otherwise}
\end{cases} \\
m \ll n & := \begin{cases} 
n m & \text{if } n :: \alpha \to \beta, \ m :: \alpha \\
\lambda k. m (\lambda x. n (\lambda f. k (x \parallel f))) & \text{otherwise}
\end{cases} \\
\end{align*}
\]

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<thead>
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<th>Item</th>
<th>Type</th>
<th>Denotation</th>
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<tr>
<td>rabbit</td>
<td>(e \to t)</td>
<td>rab</td>
</tr>
<tr>
<td>hat</td>
<td>(e \to t)</td>
<td>hat</td>
</tr>
<tr>
<td>in</td>
<td>(e \to e \to t)</td>
<td>in</td>
</tr>
<tr>
<td>some_(u)</td>
<td>((e \to \mathbb{D}_t) \to \mathbb{K}_e)</td>
<td>(\lambda\text{ckg. } \bigcup {k x g' \mid x \in \mathcal{D}_e, \langle T, g'\rangle \in c x g^{u \rightarrow x}})</td>
</tr>
<tr>
<td>the_(u)</td>
<td>(\mathbb{K}_{(e \to \mathbb{D}_t)} \to \mathbb{K}_e)</td>
<td>(\lambda\text{kg. } 1_u (k \text{some}_u) g)</td>
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| 1_\(u\) | \(\mathbb{F}_\alpha\) | \(\lambda\text{mg. } \begin{cases} 
G & \text{if } |G_v| = 1, \text{ where } G = mg \\
G_u = \{g u \mid \langle \cdot, g \rangle \in G\} & \text{otherwise}
\end{cases}\) |
\[
[\text{the rabbit in the hat}] =
\]

\[
\left( \frac{1_u \, []}{\text{some}_u (\lambda x. \, [])} \right) \parallel \left( \frac{ [] \, \parallel \, []}{\text{rab} \parallel \, \text{in}} \right) \parallel \left( \frac{1_v \, []}{\text{some}_v (\lambda y. \, [])} \right)
\]

\[
\Rightarrow \downarrow \downarrow \sim \rightarrow
\]

\[
\left( \frac{1_v \, []}{\lambda g. \cup \{ [ ] g^{\nu \mapsto y} \mid \text{hat} \, y \}} \right)
\]

\[
\Rightarrow \downarrow \downarrow
\]

\[
\left( \frac{1_u (1_v \, [])}{\text{some}_u (\lambda xg. \cup \{ [ ] g^{\nu \mapsto y} \mid \text{hat} \, y \})} \right) \parallel \left( \frac{1_u (1_v \, [])}{\lambda g. \cup \{ [ ] g^{\nu \mapsto y} \mid \text{hat} \, y, \, \text{rab} \, x, \, \text{in} \, x \, y \}} \right)
\]

\[
\Rightarrow \downarrow
\]

\[
\sim \rightarrow \left( 1_u \left( 1_v \left( \lambda g. \{ [x, g^{\nu \mapsto y}] \mid \text{hat} \, y, \, \text{rab} \, x, \, \text{in} \, x \, y \} \right) \right) \right)^* \]

\[
\Rightarrow \frac{\lambda g. \, [ ] g^{\nu \mapsto y}}{x}, \text{ where } x = ix: \text{hat}. \ \exists y. \text{rab} \, y \land \text{in} \, x \, y, \ y = iy: \text{rab}. \ \text{in} \, x \, y
\]