Universal quantification as iterated conjunction

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Abstract

I analyze distributive universal quantifiers like 'each' and 'every' in terms of iterated dynamic update. I argue that this minor adjustment to standard dynamic setups has at least two empirical advantages. First, because information flows forward through the universal computation, anaphoric elements can assume "quantifier-internal" interpretations [1]. Second, because conjunction is usually analyzed as relation composition over input and output structures, the emerging representations are in a sort of disjunctive normal form that facilitates "functional" readings of indefinites. Following [13], I suggest that these two phenomena are closely related, and argue that the current approach which generates the two readings via the same compositional mechanism is simpler, more general, and more empirically adequate than the alternatives.

1 Introduction

In many dynamic frameworks for natural language semantics, sentences denote relations over some sort of data structure. For instance, several prominent fragments in the compositional wake of DRT define sentence meanings in terms of constraints on pairs of (sets of) assignment functions, including DPL [6], CDRT [9], and PCDRT [1]. In most of these frameworks, sentential conjunction is modeled as relation composition: the meaning of $\phi \wedge \psi$ is a relation between input and output structures i and j just in case there is an intermediate structure k such that k is a possible output of ϕ evaluated at i, and j is a possible output of ψ evaluated at k. This composition is sometimes called dynamic conjunction because the intermediate structure k acts as an emissary between the conjuncts, transmitting anaphoric and truth-conditional information along the discourse gradient from ϕ to ψ.

In what follows, I will argue that distributive universal quantification consists in iterated applications of this basic sequencing operation, just as the classical universal quantifier $\forall$ generalizes static Boolean conjunction. A sentence like (1a) will be analyzed in terms of the sequence in (1b), assuming the relevant students are Mary, John, and Bill.

(1) a. Every student read a book

b. $[\![$Mary read a book$]\!]$; $[\![$John read a book$]\!]$; $[\![$Bill read a book$]\!]$

Given (i) the well-known algebraic connection between Boolean conjunction and universal quantification, (ii) the common treatment of sentential conjunction as sequential update (i.e. relation composition) in dynamic semantics, and (iii) the cross-linguistic tendency for conjunctive coordinations to be order-sensitive [14], I take it that this analysis is on plausible theoretic ground. More importantly, it delivers a couple of empirical patterns that have caused a good deal of grief for compositional semantics. First, the iterated update strategy paves the way for a uniform treatment of "internal" uses of adjectives of comparison, as in (2a). Second, when the nuclear scope contains a source of nondeterminism — typically a disjunction or indefinite DP — the strategy generates effectively functional discourse representations, of the form sketched

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in (2b). And in any framework that identifies semantic nondeterminism as the *vis viva* behind exceptional scope and exceptional binding [4], the iterated update strategy also provides an explanation for the surprising pair-list reading of sentences like (2c).

(2) a. Every year I buy {a different, a new, another, a faster} computer

b. Each guest brought a certain dish
$= \exists f : \mathsf{guest} \to \mathsf{dish} \,.\, \forall x \in \mathsf{guest} \,.\, \mathsf{brought}\,(f x)\, x$

c. If each guest brought a certain dish, the party was surely a success [11]: (23)
$= \exists f : \mathsf{guest} \to \mathsf{dish} \,.\, (\forall x \in \mathsf{guest} \,.\, \mathsf{brought}\,(f x)\, x) \Rightarrow \mathsf{success\ the.party}$

To formalize the iterated update approach, I will present a fragment in the style of Dekker's [5] Predicate Logic with Anaphora. Because of the way discourse referents accumulate monotonically over the course of a computation, the stacks of PLA make it particularly clear how the relevant functional objects take shape in the dynamic contrail of iterated conjunction. The next section lays out the relevant parts of this PLA-ish system, and Section 3 focuses the technique on the empirical issues highlighted in (2). Section 4 discusses the coverage of the procedure, and compares it to some of the alternative analyses on the market.

2 Predicate Logic with Anaphora, e.g.

As presented in [5], PLA is an update logic in which formulas are interpreted with respect to the usual model and variable assignment parameters, as well as sets of stacks that record anaphoric possibilities. To bring out the resemblance to other compositional dynamic treatments, I recast PLA in familiar Montagovian terms.[1]

To this end, sentences denote relations over stacks, or equivalently, functions from stacks to sets of stacks, of type $p \equiv sst$. Dynamically charged items modify their input stacks. Predicates, by and large, simply perform tests on the values of bound variables.

(3)

Item	*Type*	*Denotation*
John	$(ep)p$	$\lambda Q s \,.\, Q\, \mathsf{j}\, (s \cdot \mathsf{j})$
book	ep	$\lambda x s \,.\, \{s \mid \mathsf{book}\, x\}$
read	$((ep)p)ep$	$\lambda \mathcal{Q} y s \,.\, \mathcal{Q}\, (\lambda x s' \,.\, \{s' \mid \mathsf{read}\, x\, y\})\, s$
a	$(ep)(ep)p$	$\lambda P Q s \,.\, \bigcup \left\{ Q\, x\, (s' \cdot x) \;\middle|\; s' \in P\, x\, s \right\}$

Here, the name 'John' denotes a dynamic generalized quantifier. It passes the entity corresponding to John into its continuation, along with an updated discourse stack, now listing j as its most recent (final) element. 'Book' tests an individual for book-hood; if it passes, the input stack is the only thing returned unmodified, but if it fails, the input stack is discarded and the empty set returned instead. The transitive verb 'read' has been argument raised for simplicity. Like 'book', it fails quietly when its arguments do not stand in the appropriate relation, and returns just its input when they do.

These three values are *deterministic*, in the sense that when fed an input, they return at most one output. The indefinite article, on the hand, is in general *nondeterministic*. At a single input, the indefinite generates at least as many updated stacks as there are witnesses of its restrictor

[1] This re-coding of PLA follows that in [4]. The notational conventions are, I hope, mostly standard: Model-theoretic entities/relations/etc. are given in $\mathsf{sans\text{-}serif}$. Variables are in math-italics x, y, z, etc., with the usual types (x-z of type e; P, Q of type ep; s, s' for stacks). Function application is left associative; type descriptions are right associative; parentheses are omitted, except where necessary for grouping. If $s = [s_0, \ldots, s_n]$, and $i \geq 0$, then s_i identifies the *ith* element from the beginning of s; if $i < 0$, then s_i picks out the *ith* element from the end of s. $s \cdot x = [s_0, \ldots, s_n, x]$; and $s \cdot s' = [s_0, \ldots, s_n, s'_0, \ldots s'_{n'}]$. The interpretation brackets $[\![\cdot]\!]$ map each item onto its denotation, where $[\![\mathrm{X} \ldots \mathrm{Z}]\!] = [\![\mathrm{X}]\!] \ldots [\![\mathrm{Z}]\!]$. $\lambda x_0 \ldots x_n \,.\, M$ is short for $\lambda x_0 \ldots \lambda x_n \,.\, M$.

(more if the restrictor contains indefinites of its own). Then each of these updated stacks is threaded through the nuclear scope, where it is potentially subjected to further manipulation, as in (4c). In this fashion, the dynamic effects of the restrictor and nuclear scope are crossed, so that the final denotation of (4d) is a function from an input stack to a set of extending output stacks, one for each pair in $\mathsf{man} \times \mathsf{book}$.

(4) a. $[\![\text{a book}]\!] = \lambda Qs\,.\, \bigcup \big\{ Q\,x\,(s' \cdot x) \;\big|\; s' \in \{s \mid \mathsf{book}\,x\} \big\} = \lambda Qs\,.\, \bigcup \big\{ Q\,x\,(s \cdot x) \;\big|\; \mathsf{book}\,x \big\}$

b. $[\![\text{read [a book]}]\!] = \lambda ys\,.\, \bigcup \big\{ \{s \cdot x \mid \mathsf{read}\,x\,y\} \;\big|\; \mathsf{book}\,x \big\} = \lambda ys\,.\, \big\{ s \cdot x \;\big|\; \mathsf{book}\,x \wedge \mathsf{read}\,x\,y \big\}$

c. $[\![\text{John [read [a book]]}]\!] = \lambda s\,.\, \big\{ s \cdot \mathsf{j} \cdot x \;\big|\; \mathsf{book}\,x \wedge \mathsf{read}\,x\,\mathsf{j} \big\}$

d. $[\![\text{A man [read [a book]]}]\!] = \lambda s\,.\, \big\{ s \cdot y \cdot x \;\big|\; \mathsf{book}\,x \wedge \mathsf{man}\,y \wedge \mathsf{read}\,x\,y \big\}$

This last point is important. The indefinite article multiplies nondeterminism in its restrictor and nuclear scope by essentially *composing* the latter with the former. In fact, as indicated in Section 1, this is the general strategy for conjoining propositional meanings in dynamic systems. Thus the generic entry for sentential 'and' looks very much like that for 'a', but with a simpler type.[2] Both operators are in a sense prepared for nondeterministic arguments, and both respond by distributing the respective possibilites pointwise. So if there are several ways of making A true (in the sense that there are several witnesses for some nondeterministic source inside of it), and there are several ways of making B true, then saying 'A and B' amounts to saying that *some* combination of an A verifier and a B verifier is true. In other words, relation composition has disjunctive normal form in its bones; it turns '(A_0 or . . . or A_i) and (B_0 or . . . or B_j)' into '($A_0 \wedge B_0$) or ($A_0 \wedge B_1$) or . . . or ($A_i \wedge B_j$)'.

(5)

Item	*Type*	*Denotation*
and	*ppp*	$\lambda pq\,.\, p\,;q \;\equiv\; \lambda pqs\,.\, \bigcup \big\{ q\,s' \;\big|\; s' \in p\,s \big\}$
every	$(ep)(ep)p$	$\lambda PQs\,.\, \mathbin{;}\!\big\{ \lambda s'\,.\, Q\,x\,(s' \cdot x) \;\big|\; P\,x\,s \neq \emptyset \big\}\, s$,
		where $\mathbin{;}\!\{A_0, A_1, \ldots, A_n\} \equiv A_0\,; A_1\,; \ldots\,; A_n$

The semantics of 'every', as promised, folds the compositional procedure embodied by 'and' over a set of propositions built by mapping the restrictor over the nuclear scope.[3,4] 'Every' and 'a' have much in common here, as do 'and' and 'or' ($= \lambda pqs\,.\, \bigcup\{p\,s,\; q\,s\}$), of which they are generalizations. Both determiners can be seen as building a Cartesian matrix of possible computational threads from the crossed witness sets of their restrictors and scopes, but they differ in what they do with that matrix. The indefinite article does almost nothing, except reduce the dimensionality of the resulting set of sets, in accordance with its type. The universal determiner, however, *sequences* the rows of that matrix, each of which corresponds to a dynamic proposition with a single point of nondeterminism (in the simplest case).

Take the sentence in (4d) for example, and its universal counterpart in (6) (which is α-equivalent to the $\lambda s\,.\, \mathbin{;}\!\{\ldots\}\,s$ entry given in (5)).

[2] Note that this is just the usual $[\![\phi \text{ and } \psi]\!] = \lambda ss''\,.\,\exists s'\,.\, s[\phi]s' \wedge s'[\psi]s''$ dynamic denotation for 'and' translated into set-theoretic terms.

[3] Technically, because ; is not in general commutative, the semantics must make a choice about the order in which the elements of the restrictor should be evaluated. As we will see in Section 3, this choice usually does not make any difference for the resulting truth conditions, so I assume it is random. When it does make a difference, I assume it is determined by context and world knowledge.

[4] Though this entry for 'every' will pass information from one restrictor element to another, it will not pass along information from the restrictor to the scope. This simplification makes it easier to think about functional readings and internal 'different', but it means that donkey anaphora out of DP is out of the question. [3] provides a version of this universal in a monadic semantics that feeds information through both semantic dimensions, though the empirical facts concerning 'different' donkey sentences are entirely uncharted.

$$\text{(6)}\ [\![\text{Every man [read [a book]]}]\!] = \mathop{;}\left\{\lambda s' .\, (\lambda y s .\, \{s \cdot x \mid \mathsf{book}\, x \wedge \mathsf{read}\, x\, y\})\, y\, (s' \cdot y) \;\middle|\; \mathsf{man}\, y\right\}$$
$$= \mathop{;}\left\{\lambda s' .\, \{s' \cdot y \cdot x \mid \mathsf{book}\, x \wedge \mathsf{read}\, x\, y\} \;\middle|\; \mathsf{man}\, y\right\}$$

In both sentences, we are dealing with the set of propositions {John read a book, Dave read a book, Fred read a book, . . . }, one for each man. The indefinite article is content to merely pass in the input stack to each alternative *in parallel*, and to collect the results with a big $\bigcup$. But the universal runs the sentences off *in sequence*, as sketched in (7). This is just the semantic reflex of taking 'a' to generalize 'or' and 'every' to generalize 'and'.

But then we are in the situation mentioned earlier in which we are composing several nondeterministic propositions. The result is again in a kind of disjunctive normal form. It is true iff a suitable output stack can be found, i.e. iff each man can be paired with a book that he read. If any of the men in fact did not read any books, then the particular proposition that he is responsible for (e.g. 'Fred read a book') will be equivalent to $\lambda s' .\, \{\,\}$, the dynamic analog of falsity. The emptiness will inevitably swallow the entire update.

$$\text{(7)}\quad \begin{array}{c} \left(\lambda s' .\, \{s' \cdot \mathsf{m}_0 \cdot x \mid \mathsf{book}\, x \wedge \mathsf{read}\, x\, \mathsf{m}_0\}\right) ; \\ \left(\lambda s' .\, \{s' \cdot \mathsf{m}_1 \cdot x \mid \mathsf{book}\, x \wedge \mathsf{read}\, x\, \mathsf{m}_1\}\right) ; \\ \vdots \\ \left(\lambda s' .\, \{s' \cdot \mathsf{m}_n \cdot x \mid \mathsf{book}\, x \wedge \mathsf{read}\, x\, \mathsf{m}_n\}\right) \end{array} \quad\rightsquigarrow\quad \lambda s .\, \left\{ \begin{array}{c} s \cdot \mathsf{m_0} \cdot \mathsf{b_{00}} \cdot \mathsf{m_1} \cdot \mathsf{b_{10}} \cdot \ldots \cdot \mathsf{m_n} \cdot \mathsf{b_{n0}} \\ s \cdot \mathsf{m_0} \cdot \mathsf{b_{00}} \cdot \mathsf{m_1} \cdot \mathsf{b_{10}} \cdot \ldots \cdot \mathsf{m_n} \cdot \mathsf{b_{n1}} \\ \vdots \\ s \cdot \mathsf{m_0} \cdot \mathsf{b_{00}} \cdot \mathsf{m_1} \cdot \mathsf{b_{10}} \cdot \ldots \cdot \mathsf{m_n} \cdot \mathsf{b_{ni}} \\ \vdots \\ s \cdot \mathsf{m_0} \cdot \mathsf{b_{0i}} \cdot \mathsf{m_1} \cdot \mathsf{b_{1j}} \cdot \ldots \cdot \mathsf{m_n} \cdot \mathsf{b_{nk}} \end{array} \right\}$$

3 Applications

3.1 Internal Adjectives

The application of iterated conjunction to internal adjectives is immediate. The reason 'different' or 'new' or 'faster' can seem to straddle multiple elements of a universal restrictor is that the information from early evaluations feeds forward to later evaluations. Consider (2a) again. At each year, we iterate again through the VP, selecting as we go a witnessing computer for each potential branch of the update. Once a particular computer has been selected, it is recorded on that branch's stack. At the next iteration, every update to the branch will be able to see which computers have already been selected to witness the previous years' updates. This means that all 'different', e.g., has to do is restrict its complement's extension to those entities not already on the relevant stack. Likewise, 'faster' simply restricts its complement to those entities that are faster than the next fastest entity on the input stack.

(8)

Item	*Type*	*Denotation*
different	$(ep)ep$	$\lambda P x s .\, \left\{s' \;\middle|\; s' \in P\, x\, s \;\wedge\; x \notin s'\right\}$
faster	$(ep)ep$	$\lambda P y s .\, \left\{s' \;\middle|\; s' \in P\, x\, s \;\wedge\; \mathsf{speed}\, x > \max\{\mathsf{speed}\, u \mid u \in s \;\wedge\; P\, u\, s \neq \emptyset\}\right\}$

(9) presents an internally modified analog of (1a). As the derivation unfolds, one by one the assembled propositions contribute a student and a book to each of the potential update stacks. Crucially, on any given thread of the update, the new book must be different from the books already witnessing the VP on behalf of the other individuals. So if we are at, say, John's loop through the VP and we cannot find a stack to contribute to (either because John didn't read any books, or because he read a subset of the books that Mary did), then the entire update fails. If, however, there is any pairing of books to students in such a way that no book is repeated, then the update succeeds. Ultimately, all such injective pairings are retained as potential outputs of the sentence.

(9) a. $[\![\text{a [different book]}]\!] = \lambda Q s.\, \bigcup \{ Q\, x\, (s \cdot x) \mid \mathsf{book}\, x \wedge x \notin s \}$

b. $[\![\text{John [read [a [different book]]]}]\!] = \lambda s.\, \{ s \cdot \mathsf{j} \cdot x \mid \mathsf{book}\, x \wedge x \notin s \wedge \mathsf{read}\, x\, \mathsf{j} \}$

c. $[\![\text{Every student read a different book}]\!] =$
$[\![\text{Mary read a different book}]\!]$; $[\![\text{John read a different book}]\!]$; $[\![\text{Bill read a different book}]\!]$

$$\begin{array}{l} (\lambda s'.\, \{ s' \cdot \mathsf{m} \cdot x \mid \mathsf{book}\, x \wedge x \notin s' \wedge \mathsf{read}\, x\, \mathsf{m} \}) \,; \\ (\lambda s'.\, \{ s' \cdot \mathsf{j} \cdot x \mid \mathsf{book}\, x \wedge x \notin s' \wedge \mathsf{read}\, x\, \mathsf{j} \}) \,; \\ (\lambda s'.\, \{ s' \cdot \mathsf{f} \cdot x \mid \mathsf{book}\, x \wedge x \notin s' \wedge \mathsf{read}\, x\, \mathsf{f} \}) \end{array} \rightsquigarrow \lambda s.\, \left\{ \begin{array}{c|l} s \cdot \mathsf{m} \cdot x & x, y, z \in \mathsf{book}, \\ \cdot\ \mathsf{j} \cdot y & \langle \mathsf{m}, x \rangle, \langle \mathsf{j}, y \rangle, \langle \mathsf{f}, z \rangle \in \mathsf{read}, \\ \cdot\ \mathsf{f} \cdot z & x \notin s, \quad y \notin s \cdot \mathsf{m} \cdot x, \\ & z \notin s \cdot \mathsf{m} \cdot x \cdot \mathsf{j} \cdot y \end{array} \right\}$$

The same procedure generates internal readings for morphologically comparative adjectives, as well as adjectives that depend on an implicit ordering source, like 'new'. The only difference is that since these adjectives denote *asymmetric* relations, the resulting truth conditions depend on the order in which the elements are evaluated. For this reason, internal comparatives are only natural when their distributor's restrictor is either inherently ordered — usually by time — or when some ordering of the restrictor elements is recoverable from the context. For instance, (10) sketches the derivation of the comparative version of (2a). The only stacks that survive the update are those that can be incrementally updated with a sequence of computers that I bought in consecutive years, each faster than the ones preceding it.

(10) a. $[\![\text{[In 2010], [I [bought [a [faster computer]]]]}]\!] =$
$\lambda s.\, \{ s \cdot \mathsf{2010} \cdot x \mid \mathsf{comp}\, x \ \wedge\ x > \max\{ \mathsf{speed}\, u \mid \mathsf{comp}\, u \ \wedge\ u \in s \} \}$

b. $[\![\text{Every year, I bought a faster computer}]\!] =$
$[\![\text{In 2009, I bought a faster computer}]\!]$; $[\![\text{In 2010, I bought a faster computer}]\!]$; ...

c. $$\lambda s.\, \left\{ \begin{array}{l|l} s \cdot \mathsf{2009} \cdot x & x, y, z, \dots \in \mathsf{comp}, \quad \langle \mathsf{j}, x, \mathsf{2009} \rangle, \langle \mathsf{j}, y, \mathsf{2010} \rangle, \langle \mathsf{j}, z, \mathsf{2011} \rangle, \dots \in \mathsf{bought}, \\ \quad \cdot\ \mathsf{2010} \cdot y & x > \max\{ \mathsf{speed}\, u \mid \mathsf{comp}\, u \ \wedge\ u \in s \}, \\ \quad \cdot\ \mathsf{2011} \cdot z & y > \max\{ \mathsf{speed}\, u \mid \mathsf{comp}\, u \ \wedge\ u \in s \cdot \mathsf{2009} \cdot x \}, \\ \quad \cdot\ \dots & z > \max\{ \mathsf{speed}\, u \mid \mathsf{comp}\, u \ \wedge\ u \in s \cdot \mathsf{2009} \cdot x \cdot \mathsf{2010} \cdot y \},\ \dots \end{array} \right\}$$

3.2 Functional Readings

Returning to the example in (6), we said that 'Every man read a book' is true just in case there is a pairing of men with books such that each man read the book he is paired with. These truth conditions are equivalent to the usual $\forall \gg \exists$ variety, and are in fact exactly what we'd get by Skolemizing the indefinite and existentially closing over the resulting parameterized function, as in the gloss of (2b). But we got to this point simply by sequencing a number of nondeterministic propositions and letting the nondeterminism bubble through the conjunctions. When this happens, we are left with a fundamentally *disjunctive* kind of meaning, the sort of meaning that takes a single input stack to a (potential) multiplicity of output stacks. Each of these output stacks will correspond to a function $f : \mathsf{man} \to \mathsf{book}$ mapping each man to one of the books he read.

Following [10, 8, 11], and especially [4], I will assume that nondeterminism is at the heart of exceptional scope; indefinite DPs and disjunctions of any category can outscope any chunk of their compositional context, while quantificational DPs and conjunctions of any category are effectively clause-bounded.[5] If this is right, then we already have an account of the "functional

[5]The unconvinced reader is referred to [4], where the point is given theoretical teeth in a continuations-based semantics where scope-taking possibilities follow directly from semantic types. [3] implements the strategy here in that continuized grammar.

readings" of sentences like (2c).[6] What's really taking scope here are the alternative ways of satisfying the universal proposition. (2b), for example, corresponds to a meaning along the lines of "EITHER Mary brought turkey, John potatoes, and Fred casserole, OR Mary brought turkey, John casserole, and Fred potatoes, OR ...", for each possible pairing of individuals and dishes. The functional reading of (2c) then corresponds to the wide scope interpretation of this disjunction with respect to the conditional it is in: "EITHER 'Mary brought turkey, John potatoes, and Fred casserole' OR 'Mary brought turkey, John casserole, and Fred potatoes' OR ... is such that if it came true, then the party was surely a success".

It is hard to be more precise about this without an explicit theory of scope-taking, but that would take us a far afield. Nevertheless, these are exactly the truth conditions predicted by the Skolem-function gloss typically assigned to such readings, exemplified by the translation in (2c). But again, they are generated without any explicit representation of functional variables as part of the meaning of either DP.

That said, we may occasionally want to make use of the implicit functions generated by iteration, *qua* functions. For example, the little discourse in (11a) intuitively sets up a functional relationship between students and books, and then elaborates on that relationship. The second sentence makes anaphoric reference to whichever alternative happens to witness the first sentence, and when it retrieves that alternative, it retrieves it *as a function*.

One way to view this sort of functionalization on the fly is just as an iterated version of well known cases of pluralization on the fly, as in 'A man walked in, and then a woman walked in. They sat down together.' To give an example of how this might play out, the entry for 'other' in (11b) deploys a functionalizing operator $\mathcal{F}$ that replaces a (contextually-determined) subportion of the input stack with a function f by pairing off the relevant alternating entities, so that $[\mathsf{m}, x, \mathsf{j}, y, \mathsf{f}, z]$ becomes $f \equiv \{\langle \mathsf{m}, x\rangle, \langle \mathsf{j}, y\rangle, \langle \mathsf{f}, z\rangle\}$. Then 'other' returns a dynamic property true of entities that are distinct from what the function assigns to the most recent entry on the stack (which will be the individual added by the current pass through the local distributor's restrictor). In this sense, 'other' behaves exactly like a quantificationally-subordinated pronoun, in that it retrieves an anaphoric *ee*-type dependency from the discourse, and uses it to fix the reference of some bound pronominal element. (11c) gives some idea of what this looks like in action, where ${}^{\bullet}f = [x_0, f(x_0), x_1, f(x_1), \ldots]$, for $x_0, x_1, \ldots$ in the domain of f.

(11) a. Every student read a book. Then every student read another book.

b. $[\![\text{other}]\!] = \lambda P x s\,.\, \left\{ s' \;\middle|\; s' \in P\, x\, (\mathcal{F}\, s) \;\wedge\; x \neq s'_f(s'_{-1}) \right\}$

c. $\lambda s\,.\, \left\{ s \cdot {}^{\bullet}f \;\middle|\; \begin{array}{l} f : \mathsf{student} \to \mathsf{book} \\ f \subseteq \mathsf{read} \end{array} \right\} \;;\; \lambda s\,.\, \left\{ s \cdot {}^{\bullet}g \;\middle|\; \begin{array}{l} g : \mathsf{student} \to \mathsf{book}, \quad g \subseteq \mathsf{read} \\ g(\mathsf{m}) \neq f((s \cdot \mathsf{m})_{-1}), \\ g(\,\mathsf{j}\,) \neq f((s \cdot \mathsf{m} \cdot g(\mathsf{m}) \cdot \mathsf{j})_{-1}), \\ g(\,\mathsf{f}\,) \neq f((s \cdot \mathsf{m} \cdot g(\mathsf{m}) \cdot \mathsf{j} \cdot g(\mathsf{j}) \cdot \mathsf{f})_{-1}) \end{array} \right\}$

4 Discussion and Comparison

Most accounts of functional readings have appealed to Skolemized choice-functional representations of indefinite determiners (e.g., [12, 11]). To this end, an indefinite DP like 'a certain dish' is represented in the semantics as $f\, x\, \mathsf{dish}$, where the f variable, ranging over type $e(et)e$ (paramaterized) choice functions, will eventually be existentially or contextually bound, and the x variable will eventually be bound by the universal.

[6] I follow [7, 12, 11, 13] in calling the relevant truth conditions of 'every' $\gg$ 'a' sentences "functional readings", though as several of those authors have pointed out, there is good reason to keep the notion of a functional reading separate from that of the arbitrary pair-list interpretations described here. We return to the issue briefly in Section 4.

However, as both [12] and [11] point out, the choice-functional analysis dramatically overgenerates in non-upward-entailing contexts. (12), for example, is falsely predicted to mean that no student read every book. They both conclude, then, that choice function variables only ever refer to "natural functions", which rules out the arbitrary potential functional witnesses that render the logical form of (12) so weak.

(12) No student read a book that I recommended
$\neq \exists f \,.\, \neg\exists x \in \mathsf{student} \,.\, \mathsf{read}\, x\, (f\, x\, \mathsf{book.I.recommended})$

Yet, as [13] argues, this puts too strong a bind on the potential functions that may emerge from 'every' $\gg$ 'a' configurations, and doesn't really rule out the improper readings of other quantifiers in the same positions. [11] makes a similar point regarding the difference between pair-list and "natural functional" answers to questions. (13) can be answered by naming a function that intensionally maps guests to dishes, or by specifying the contents of such a mapping in an arbitrary way. But the same question with any non-distributive-universal quantifier (14) can only be answered with the former.

(13) Which dish did every guest bring?
a. His favorite
b. Al pasta; Bill salad; Carl pudding

(14) Which dish most/several/no guests bring?
a. Their favorite
b. # Al pasta; Bill salad; Carl pudding

[13] observed that the same contrast is evident in conditionals like (2c). The sentences in (15a) and (15b) lack any hint of the arbitrary pair-list assignment of guests to dishes that underlies the truth conditions of (2c). That is, (15a) doesn't mean that for some pairing of guests to dishes, if any two guests brought their dish, the party probably went well; and (15b) doesn't mean that if no guest brought their assigned dish, the party was a flop. If they have any functional reading at all, it depends on 'a certain dish' being interpreted as some sort of functional definite description, á la 'the one his mother suggested'. What's more, no restriction on "naturalness" will suffice to rule out the functional readings of (15), since precisely the same functions verify the available reading of (2c).

(15) a. If two guests brought a certain dish, the party was probably a success
b. If no guest brought a certain dish, the party was surely a failure

[2] presents a number of other problems for generalized choice-functional analyses of exceptionally scoping indefinites. The authors suggest instead that functional readings instantiate a kind of quantificational subordination to a contextually salient association. However, like the choice-functional approaches, this doesn't explain the general ability of 'every'/'each' to support such readings, as in (2c), and the general inability of all other quantifiers to do so, as in (15). What's more, the existential force of the quantification over functional witnesses can take intermediate scope beneath other operators, which is unexpected if the witness is due to some kind of contextual anaphora. In fact, (16) has a reading on which the pair-list that witnesses the embedded 'hope' clause donkey-binds the propositional anaphor in the consequent: whenever Mary has a specific wishlist about the dishes people will bring, *whatever it happens to be*, I have the same wishlist. It's hard to see how this could be derived from a deictic dependency.

(16) Whenever Mary hopes that everyone brings a certain dish, I hope so too

The lesson from functional readings seems to be that something special happens when plain indefinites are within the nuclear scope of universal distributors. Then, and only then, can arbitrary nondeterministic pair-list associations emerge between the two quantificational restrictors. As a result, all of the attempts to locate the expressive force of suc readings in the ability of the indefinite determiner to go proxy for a Skolemized choice function are bound to overgenerate, because they pay no attention to the special role of the universal.

In contrast, theories of internal 'different' typically rely on a souped-up universal because it is well-established that singular comparatively modified NPs can only be interpreted "internally" in the scope of a genuinely distributive universal quantifier. So [1], for instance, develops a dynamic semantics in which 'every' temporarily generates two independent update streams, which are simultaneously universally quantified over. 'Different' bridges the anaphoric gap between the quantificational channels, in the same way that it might compare items across sentences. This is very much in the same spirit as what I've proposed here, but with iterated conjunction, there's no need for dual quantification. The universal quantifier just plows through its restrictor, accumulating referents as it goes.

5 Conclusion

That "internal" and "functional" readings have the same very particular syntactic distribution should be a clue that they are derived from the same underlying mechanism. While polyadic quantification generates appropriate truth conditions for internal uses of comparative adjectives (at least, for symmetric relations like 'same' and 'different'), it doesn't give us any leg up on the emergence of exceptionally scoping/binding functional witnesses. And while Skolemized choice functions generate appropriate truth conditions for 'every' $\gg$ 'a' configurations, they generate inappropriate truth conditions for all other configurations.

The analysis presented here, on the other hand, derives both kinds of readings as by-products of iterated conjunction, in any semantics that treats conjunction as relation composition. This, as [4] stresses, is standard in dynamic setups. As a result, the approach is both more general and more conservative than the choice-functional and polyadic solutions currently in the literature.

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