Split-scope definites

How ‘the’ can mean two things at once

Dylan Bumford

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New York University

Definite description

Wisdom: 'the NP' refers to the single salient 'NP' in the context

$$[\![\text{the hat}]\!] = x, \text{ where } x \text{ is the unique relevant hat}$$

Proposal: Definite determination split into two subprocesses.

$$[\![\text{the hat}]\!] = \textbf{one}\,(\cdots(\textbf{some}\;\text{hat}))$$

When things intervene, 'the hat' may end up one among many

Payoffs:

- Haddock readings
- Relative superlatives
- Possibly other strange readings of quantificational adjectives
- Emerging uniformity in the theory of cardinal modification

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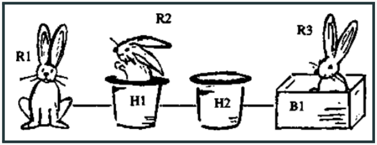
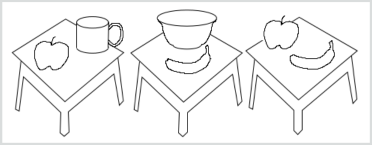


What about H2?
cf. #The hat is my favorite

(2) the table with the apple and the banana [Horacek 1995]



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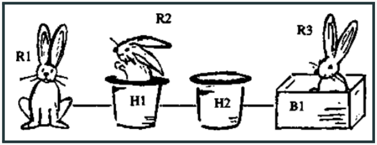

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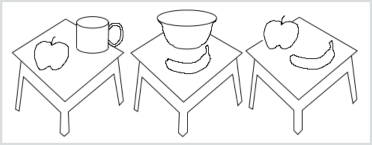

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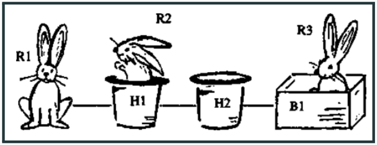

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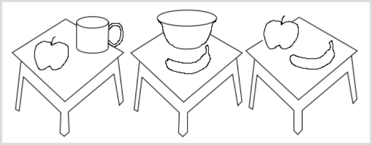

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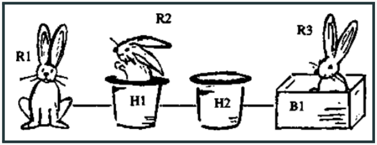

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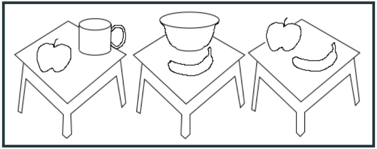

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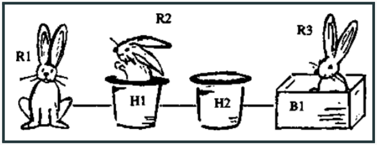

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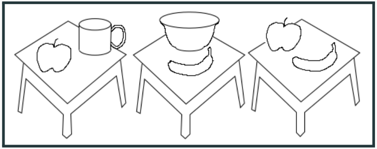

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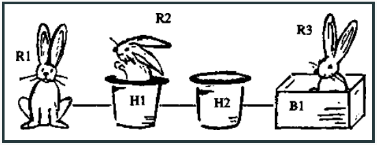

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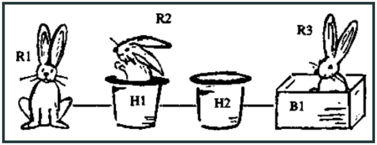

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Constraint Satisfaction Problem

x y
rabbit x
x in $\cdots$
hat y

Unique x and y satisfying these simultaneous constraints

Noncompositional. Worse, circular!

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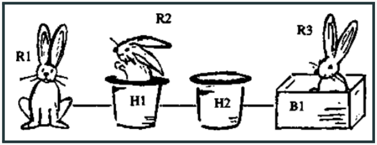

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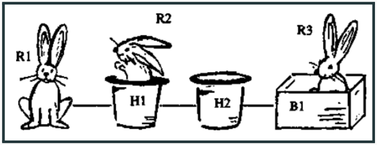

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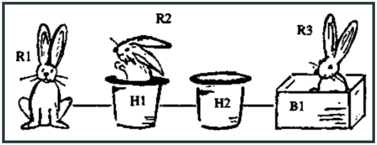

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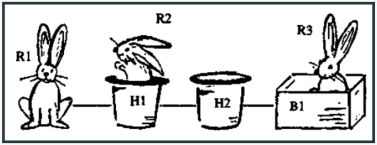

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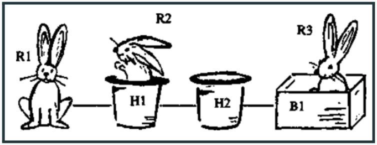

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(2) the girl who got the fewest letters [Szabolcsi 1986]

(3) a. *When was there the rabbit in the garden?

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Relative superlatives via constraint satisfaction?

(5) the rabbit in the biggest hat

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x in
hat y

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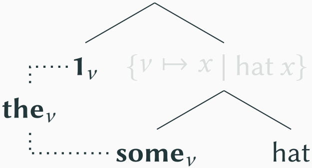
Decomposing definiteness

The basic idea: definiteness is a two-step process

$$\begin{cases} G & \text{if } \left|\{g\,v \mid g \in G\}\right| = 1 \\ \# & \text{otherwise} \end{cases}$$



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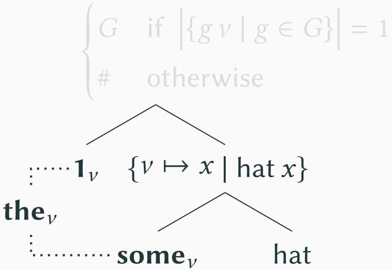

Dynamic Semantics

- Denotations are sets of assignments
- Indefinites introduce *nondeterministic* referents

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$$\begin{cases} G & \text{if } \left|\{g\,v \mid g \in G\}\right| = 1 \\ \# & \text{otherwise} \end{cases}$$

$\mathbf{1}_v$ $\{v \mapsto x \mid \text{hat}\ x\}$

$\mathbf{the}_v$

$\mathbf{some}_v$ hat

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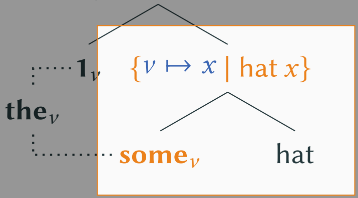

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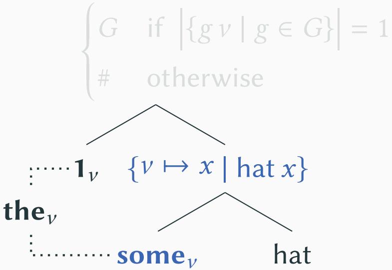

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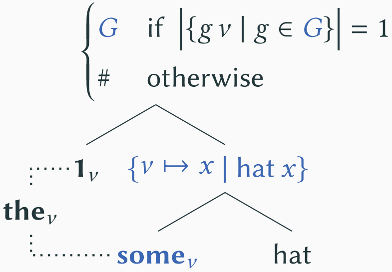

Teasing the pieces apart

(6) the [rabbit in the hat]

(7) the [rabbit in the biggest hat]

Teasing the pieces apart

some **some**

(6) the [rabbit in the hat]

one **one**

(7) the [rabbit in the biggest hat]

Teasing the pieces apart

(6) **one** [**some** rabbit in [**one** [**some** hat]]]

(7) the [rabbit in the biggest hat]

Teasing the pieces apart

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one **one biggest**

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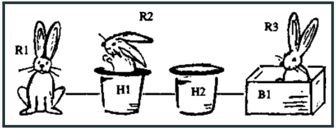

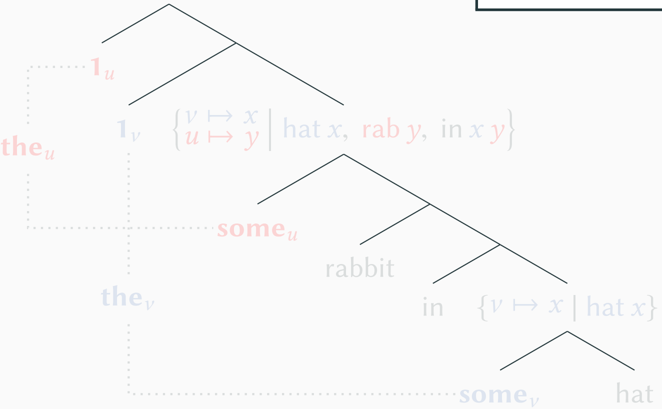

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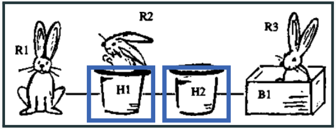

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$\mathbf{the}_u$

$\mathbf{1}_v$ $\quad \left\{ \begin{matrix} v \mapsto x \\ u \mapsto y \end{matrix} \,\middle|\, \text{hat}\; x,\ \text{rab}\; y,\ \text{in}\; x\; y \right\}$

$\mathbf{some}_u$

rabbit

$\mathbf{the}_v$

in $\quad \{ v \mapsto x \mid \text{hat}\; x \}$

$\mathbf{some}_v$ $\quad$ hat

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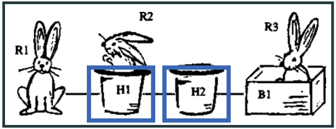

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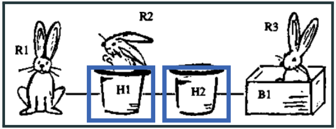

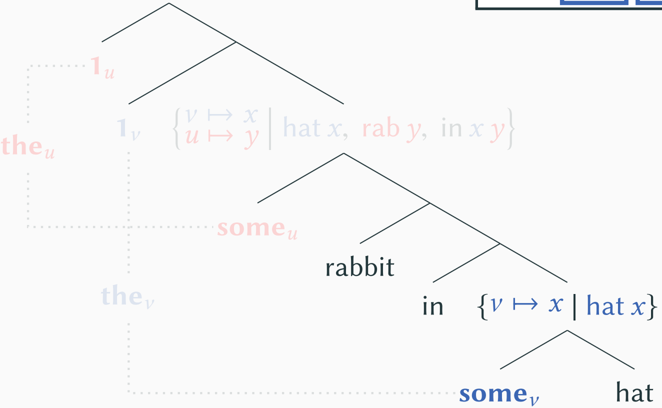

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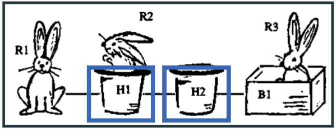

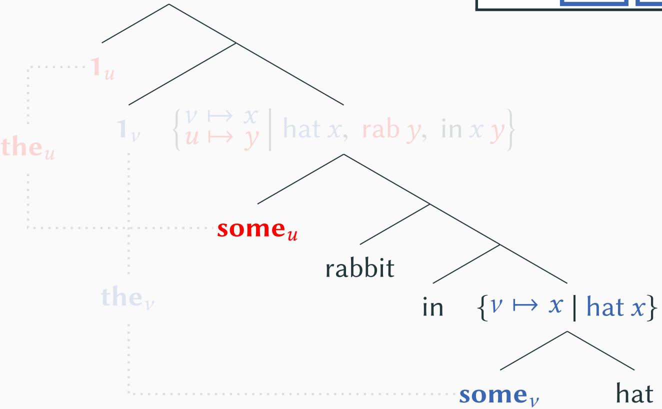

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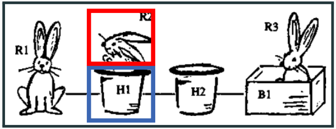

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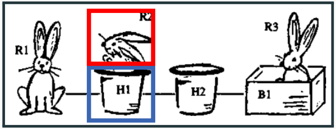

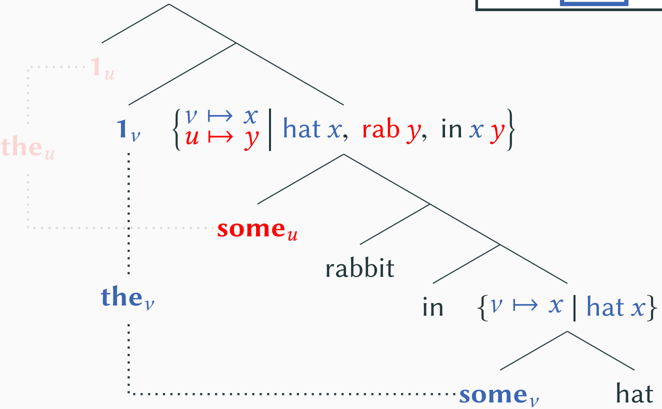

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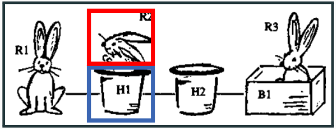

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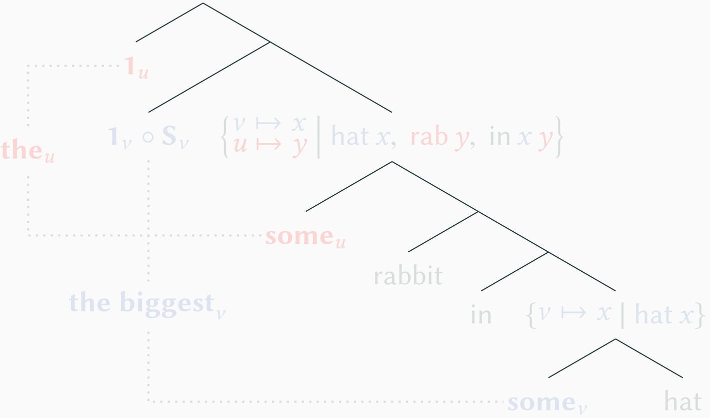
Relative superlatives: Delayed maximality filter



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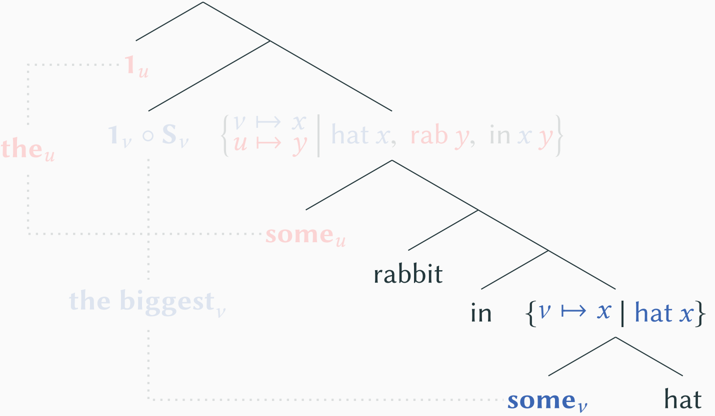

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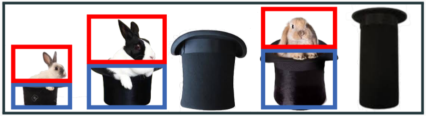

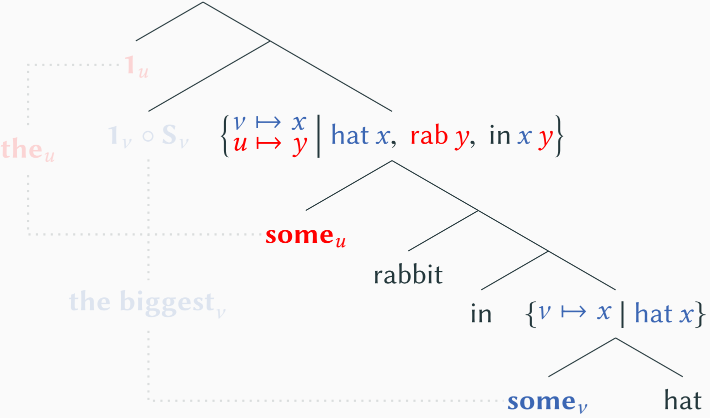

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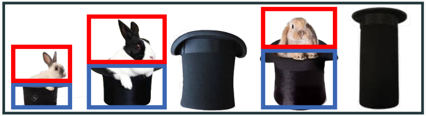

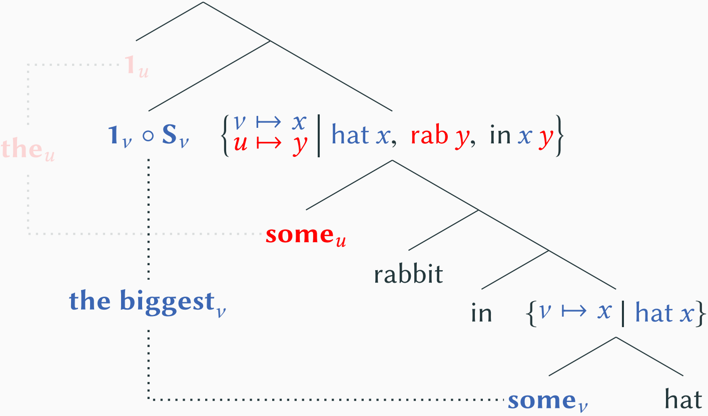

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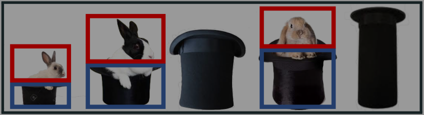


$\mathbf{1}_u$

the_u

$$\mathbf{1}_v \circ \mathbf{S}_v \quad \left\{ \begin{matrix} v \mapsto x \\ u \mapsto y \end{matrix} \,\middle|\, \text{hat}\ x,\ \text{rab}\ y,\ \text{in}\ x\ y \right\}$$

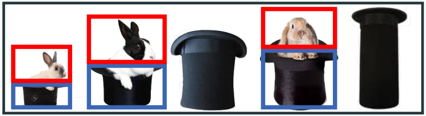
some

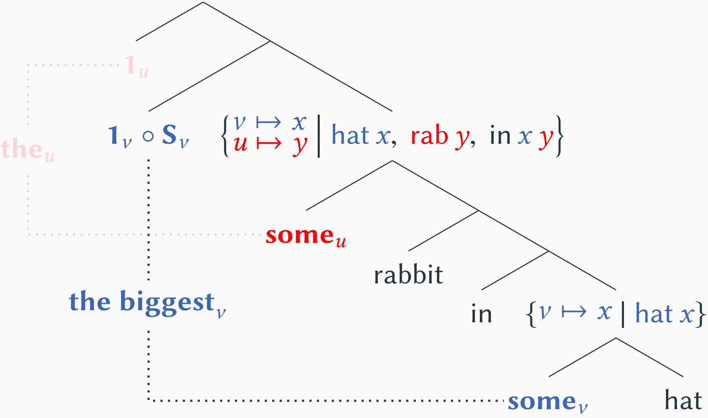
Superlative as filter

Keep only the assignments that are undominated in their choice of v

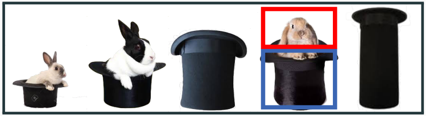
$$\mathbf{S}_v\, G := \{g \in G \mid \neg\exists g' \in G.\, g'\, v > g\, v\}$$

Relative superlatives: Delayed maximality filter





Relative superlatives: Delayed maximality filter



$\mathbf{1}_u$

$\mathbf{the}_u$

$\mathbf{1}_v \circ \mathbf{S}_v \quad \left\{ \begin{matrix} v \mapsto x \\ u \mapsto y \end{matrix} \,\middle|\, \text{hat } x,\ \text{rab } y,\ \text{in } x\ y \right\}$

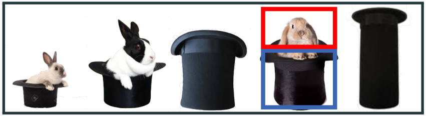
$\mathbf{some}_u$

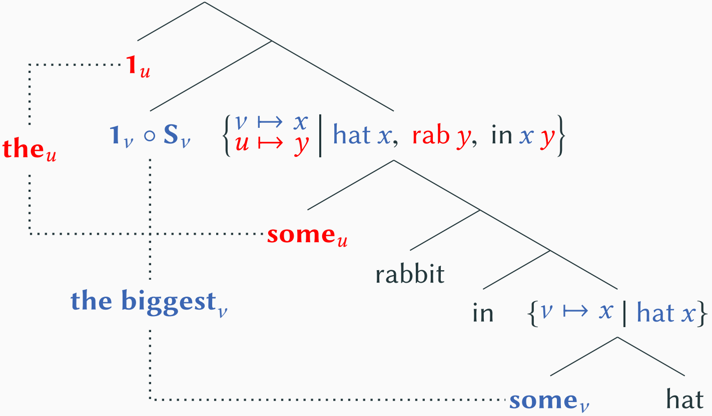
rabbit

$\mathbf{the\ biggest}_v$

in $\quad \{ v \mapsto x \mid \text{hat } x \}$

$\mathbf{some}_v$ hat




$\mathbf{1}_u$
$\textbf{the}_u$
$\mathbf{1}_v \circ \mathbf{S}_v$
$\left\{ \begin{smallmatrix} v \mapsto x \\ u \mapsto y \end{smallmatrix} \middle| \text{hat } x, \text{ rab } y, \text{ in } x\ y \right\}$
$\textbf{some}_u$
rabbit
$\textbf{the biggest}_v$
in
$\{v \mapsto x \mid \text{hat } x\}$
$\textbf{some}_v$
hat

Connections and applications: Quantificational adjectives

Range of quantificational adjectives that ride on the scope of the definite article

(8) John gave Mary the first telescope [Bylinina et al. 2014]

a. John was the first to give Mary a telescope

(9) Mary didn't score the only goal [Coppock & Beaver 2015]

a. Mary wasn't the only one to score a goal

(10) Ann read the same book yesterday and today [Barker 2007]

a. Ann read a book yesterday and a book today;
they where the same

Connections and applications: Split numerosity

And more generally, cardinality-testing denotations appear happy to take delayed action

(11) You should talk to at least three professors [Cresti 1995]
   a. You should talk to some professors; three at the least

(12) Exactly three boys saw exactly five movies [Brasoveanu 2012]
   a. Some boys saw some movies; three and five, to be exact

Zooming out: Multidimensionality in meaning

Plenty of constructions known to contribute two kinds of meaning at once

- Focus

  I gave the book to JOHN

- Conventional Implicature and presupposition

  John, a linguist, received a mysterious book

- Anaphora and discourse referent management

  A man walked in; he asked John about his book

- Alternative generation

  John either liked or hated his book; I can't remember

Scope as multidimensional meaning

- Quantification is a kind of multidimensional effect

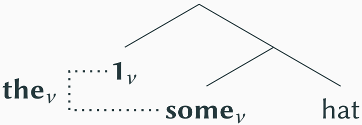
$$\frac{\text{every}_x \text{ student}}{\text{John talked to } \quad x}$$

- Definiteness is just like that, but more

$$\begin{array}{c} \text{one}_u \\ \text{some}_u \\ \text{John talked to } \quad u \end{array}$$

Conclusion

- Definiteness is semantically bipartite



- Mismatches in the execution of the parts accounts for relative readings of definites and superlatives, and possibly other quantificational adjectives

- Encourages a multidimensional view of meaning, in which different subprocesses of a denotation may act at different times on different arguments

Thanks

Thanks!

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$$m \mathbin{/\!\!/} n := \begin{cases} m\,n & \textbf{if } m :: \alpha \to \beta,\ n :: \alpha \\ \lambda k.\, m\big(\lambda f.\, n\big(\lambda x.\, k\big(f \mathbin{/\!\!/} x\big)\big)\big) & \textbf{otherwise} \end{cases}$$

$$m \mathbin{\backslash\!\!\backslash} n := \begin{cases} n\,m & \textbf{if } n :: \alpha \to \beta,\ m :: \alpha \\ \lambda k.\, m\big(\lambda x.\, n\big(\lambda f.\, k\big(x \mathbin{\backslash\!\!\backslash} f\big)\big)\big) & \textbf{otherwise} \end{cases}$$

$$m \parallel n := \begin{cases} \lambda x.\, m\,x \wedge n\,x & \textbf{if } m :: \alpha \to \beta,\ n :: \alpha \to \beta \\ \lambda k.\, m(\lambda x.\, n(\lambda f.\, k(f \parallel x))) & \textbf{otherwise} \end{cases}$$

Item	*Type*	*Denotation*
rabbit	$e \to t$	rab
hat	$e \to t$	hat
in	$e \to e \to t$	in
$\textbf{some}_u$	$(e \to \mathbb{D}_t) \to \mathbb{K}_e$	$\lambda ckg.\, \bigcup\{k\,x\,g' \mid x \in \mathcal{D}_e,\ \langle \mathbf{T}, g'\rangle \in c\,x\,g^{u \mapsto x}\}$
$\textbf{the}_u$	$\mathbb{K}_{(e \to \mathbb{D}_t) \to \mathbb{K}_e}$	$\lambda kg.\, \mathbf{1}_u\,(k\,\textbf{some}_u)\,g$
$\mathbf{1}_u$	$\mathbb{F}_\alpha$	$\lambda mg.\, \begin{cases} G & \text{if } \lvert G_v\rvert = 1, \text{ where } G = m\,g, \; G_u = \{g\,u \mid \langle \cdot, g\rangle \in G\} \\ \# & \text{otherwise} \end{cases}$

$[\![\text{the rabbit in the hat}]\!] =$

$$\left(\frac{\mathbf{1}_u\,[\,]}{\dfrac{\mathbf{some}_u\,(\lambda x.\,[\,])}{x}} \mathbin{\Big\backslash\!\!\Big\backslash} \left(\frac{[\,]}{\dfrac{[\,]}{\text{rab}}} \mathbin{\Big\|} \frac{[\,]}{\dfrac{[\,]}{\text{in}}} \mathbin{\Big/\!\!\Big/} \left(\frac{\mathbf{1}_v\,[\,]}{\dfrac{\mathbf{some}_v\,(\lambda y.\,[\,])}{y}} \mathbin{\Big\backslash\!\!\Big\backslash} \frac{[\,]}{\dfrac{[\,]}{\text{hat}}}\right)^{\Downarrow}\right)\right)^{\Downarrow\downarrow\!\uparrow} \rightsquigarrow$$

$$\left(\frac{\mathbf{1}_u\,[\,]}{\dfrac{\mathbf{some}_u\,(\lambda x.\,[\,])}{x}} \mathbin{\Big\backslash\!\!\Big\backslash} \left(\frac{[\,]}{\dfrac{[\,]}{\text{rab}}} \mathbin{\Big\|} \frac{[\,]}{\dfrac{[\,]}{\text{in}}} \mathbin{\Big/\!\!\Big/} \frac{\mathbf{1}_v\,[\,]}{\dfrac{\lambda g.\,\bigcup\{[\,]\,g^{v\mapsto y} \mid \text{hat}\ y\}}{y}}\right)\right)^{\Downarrow\downarrow\!\uparrow} \rightsquigarrow$$

$$\left(\frac{\mathbf{1}_u\,(\mathbf{1}_v\,[\,])}{\dfrac{\mathbf{some}_u\,(\lambda xg.\,\bigcup\{[\,]\,g^{v\mapsto y} \mid \text{hat}\ y\})}{\text{rab}\ x \wedge \text{in}\ y\ x}}\right)^{\Downarrow\downarrow\!\uparrow} \rightsquigarrow \left(\frac{\mathbf{1}_u\,(\mathbf{1}_v\,[\,])}{\dfrac{\lambda g.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}} \,\middle|\, \text{hat}\ y,\ \text{rab}\ x,\ \text{in}\ y\ x\right\}}{x}}\right)^{\downarrow\!\uparrow}$$

$$\rightsquigarrow \left(\mathbf{1}_u\left(\mathbf{1}_v\left(\lambda g.\,\left\{\left\langle x,\, g^{\substack{u\mapsto x\\ v\mapsto y}}\right\rangle \,\middle|\, \text{hat}\ y,\ \text{rab}\ x,\ \text{in}\ y\ x\right\}\right)\right)\right)^{\star}$$

$$\rightsquigarrow \frac{\lambda g.\,[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}}}{x},\ \textbf{where}\ x = \iota x\colon \text{hat}.\ \exists y.\ \text{rab}\ y \wedge \text{in}\ x\ y,$$
$$y = \iota y\colon \text{rab}.\ \text{in}\ x\ y$$