Linguist and Philos (2013) 36:355–369
DOI 10.1007/s10988-013-9139-z

RESEARCH ARTICLE

Association with distributivity and the problem of multiple antecedents for singular *different*

Dylan Bumford · Chris Barker

Published online: 11 October 2013

Abstract Brasoveanu (Linguist Philos 34:93–168, 2011) argues that "different" exhibits what he calls association with distributivity: a distributive operator such as "each" creates a two-part context that propagates through the compositional semantics in a way that can be accessed by a subordinate "different". We show that Brasoveanu's analysis systematically undergenerates, failing to provide interpretations of sentences such as "Every_1 boy claimed every girl read a different_1 poem", in which "different" can associate with a non-local distributive operator. We provide a generalized version of association with distributivity, implemented using de Groote's (in: Proceedings of semantics and linguistic theory XVI, 2006) continuation-based dynamic semantics. We compare our analysis with the one in Brasoveanu (2011), drawing conclusions about computational tractability, scope of indefinites, and whether it is possible or even desirable to arrive at a unified analysis of internal and external readings of "different".

Keywords Different · Same · Association with distributivity · Dynamic semantics · Continuations · Scope · Indefinites

Brasoveanu (2011) argues that certain expressions exhibit what he calls "association with distributivity" (AwD for short).

(1) a. Every boy read a different poem.
 b. The boys read a different poem.

This paper grew out of a discussion of Brasoveanu 2011 in a NYU seminar held in the spring of 2012. Thanks to insights of Neil Myler, Sofya Kasyanenko, Simon Charlow, and two anonyous referees.

D. Bumford (✉) · C. Barker
New York University, New York, NY, USA
e-mail: dbumford@gmail.com

The claim is that in (1a), *different* can be anaphorically linked to the distributivity introduced by the quantificational determiner *every*, in which case (1a) entails that no poem was read by more than one boy. In contrast, in (1b) the plural *the boys* does not introduce distributivity, which is why (1b) lacks the reading just described. Instead, (1b) only has an external reading on which *different* is anaphoric to some element outside of the sentence.

But the implementation of the AwD strategy in Brasoveanu (2011) systematically undergenerates possible interpretations.

(2) a. Every boy claimed that every girl read a different poem.
b. Every boy made the following claim: that no two girls read the same poem
c. For every boy x there is a different poem y such that x claimed every girl read y.

For instance, the Brasoveanu (2011) fragment predicts that (2a) has only one internal reading, the one paraphrased in (2b). The reason is that the formal analysis forces *different* to associate with whichever distributive operator takes narrowest scope. But native speakers report that (2a) can also have a paraphrase as in (2c), on which *different* associates with the wider-scope distributive operator.

We show one concrete way to extend the AwD approach to handle the ambiguity illustrated in (2). Although the extension provides a reasonable account of the data, it comes with a computational cost. In Brasoveanu (2011), the compositional semantics tracks twice as much information in the scope of a distributive universal quantifier as it does elsewhere in the grammar. But to provide enough flexibility for *different* to choose among several possible distributors, the amount of information that must be tracked is now linear in the number of distributive operators (though as in Brasoveanu's fragment, this additional information only persists in the scope of the distributor).

1 AwD in de Groote's continuation-based dynamic semantics

We will develop an analysis here that we intend to faithfully embody the spirit of the AwD approach, but which differs from the implementation in Brasoveanu (2011) technically in many ways, both small and large. In addition to providing a compatible account of the ambiguity in (2), having a second implementation of association with distributivity will allow us to discuss below what is essential to the approach, and what is implementation-specific.

Association with distributivity is named on analogy with association with focus. In association with focus, focus creates a two-part structured meaning consisting of a focused denotation and a background. This two-part meaning is propagated upwards, where it can be accessed by a focus-sensitive operator such as *only*. Association with distributivity works in the opposite direction, from top to bottom: a distributive operator like *every* creates a two-part context, which is passed along to arguments, where it can be accessed by a distributivity-sensitive operator such as *different*.

The backbone of the compositional analysis here will be the continuation-based dynamic semantics of de Groote (2006). In de Groote's framework, expressions are evaluated with respect to two parameters: an input context (a sequence of individuals) and a continuation (more on this shortly). To recreate the bipartite effect of AwD in this de Grootian setting, we will evaluate expressions against not one but two independent input contexts. And later, to make room for multiple potential licensors, we will trade in the pair of inputs for a list of inputs, the length of which is in principle unbounded.

Types		Variables	Examples
Entity:		x, y	**a**, **b**, **c**, …
Stack:	List of Entities	i, j	**ab**, **aac**, …
Bool:	Truth value		TRUE, FALSE
Dref:	Discourse referent (integer)	n, m, l	0, 1, 2, …
Predicate:	Dref → Proposition	P, Q	**entered**, **sat**, …
Continuation:	Stack → Stack → Bool	κ	λij . TRUE
Proposition:	Stack → Stack → Continuation → Bool	p, q	

There are three base types: entities, truth values, and discourse referents. Discourse referents are implemented as integers, so that a particular dref singles out a position, an index, in whatever sequence of individuals it is interpreted against (note that following a convention in computer science, sequence positions start at 0). As in Brasoveanu (2011), we will call a sequence of individuals a ‘stack’, and we will use the following notation:

Notation	Gloss	Example
i_n	the nth element in the stack i	$(\mathbf{abc})_0 = \mathbf{a}$
$x{:}i$	the list formed by prepending x to i	$\mathbf{a}{:}(\mathbf{abc}) = \mathbf{aabc}$
$i^{x/n}$	the list formed by inserting x in position n of i	$(\mathbf{abc})^{\mathbf{a}/2} = \mathbf{abac}$

Dynamic propositions are modeled here as update functions that need three things in order to deliver a truth value: two independent stacks, representing the context in which the proposition is evaluated, and a continuation, representing the rest of the discourse. A simple example will show the role the continuation plays in dynamic interpretation. See de Groote (2006) for motivation and additional discussion.

(3) a. Alex entered and he$_0$ sat.
b. $[\![Alex]\!] = \lambda P i j \kappa \,.\, P\, 0\, i^{\mathbf{a}/0}\, j^{\mathbf{a}/0}\, \kappa$
c. $[\![entered]\!] = \lambda n i j \kappa \,.\, i_n, j_n \in \mathbf{enter} \wedge \kappa\, i\, j$
d. $[\![and]\!] = \lambda p q \,.\, p; q,$ where $p; q \equiv \lambda i j \kappa \,.\, p\, i\, j\, (\lambda i' j' \,.\, q\, i'\, j'\, \kappa)$
e. $[\![he_n]\!] = \lambda P i j \kappa \,.\, P\, n\, i\, j\, \kappa$
f. $[\![sat]\!] = \lambda n i j \kappa \,.\, i_n, j_n \in \mathbf{sit} \wedge \kappa\, i\, j$

The essentially dynamic element here is the conjunction in (3d), which evaluates the second conjunct q with respect to context stacks that have been updated by evaluation of the left conjunct p.

In somewhat more detail: the proper name *Alex* (functioning as a generalized quantifier) pushes the individual Alex, represented here by the object **a**, onto both of the stacks in its context, and instructs the verb phrase P to look for its subject in the 0th position of the updated stacks. The predicate *entered* checks to make sure that the objects in the designated position n of the two stacks it is asked to evaluate (i and j) both have the property of entering. It then passes the stacks on to its continuation (i.e., the rest of the sentence). In this case, the continuation is the right conjunct, *he*$_0$ *sat*. The pronoun does not add any new item to the stacks, but like *Alex* tells its verb phrase where to look for its subject, in this case, once again, the 0th position. As long as the index of the pronoun is 0, the entity that sat in this discourse will be the same entity that entered. More precisely, the discourse $[\![and]\!]\ ([\![alex]\!]\ [\![entered]\!])\ ([\![he_0]\!]\ [\![sat]\!])$ will evaluate to TRUE when applied to any initial pair of stacks and the trivial continuation $\text{TRIV} = \lambda ij\,.\,\text{TRUE}$, as long as Alex entered and Alex sat.

The central idea of the AwD strategy is that distributive operators like *every* manipulate the two context stacks in a coordinated way:

(4) a. Every^0 boy entered.
b. $[\![every^n]\!] = \lambda PQij\kappa\,.\,(\forall x, y, x \neq y\,.\,P\,n\,i^{x/n}\,j^{y/n}\,\text{TRIV} \rightarrow (P\,n;\,Q\,n)\,i^{x/n}\,j^{y/n}\,\text{TRIV}) \wedge \kappa\,i\,j$
c. $[\![boy]\!] = \lambda nij\kappa\,.\,i_n, j_n \in \textbf{boy} \wedge \kappa\,i\,j$

As in Brasoveanu (2011), distributivity here requires dual quantification, that is, quantification over distinct pairs of individuals. For every choice of distinct x and y in the domain, we update the input contexts i and j by inserting x in the nth position of i, and y in the nth position of j. Then every choice of distinct x and y that satisfies the restrictor—i.e., every choice of x and y such that $P\,n\,i^{x/n}\,j^{y/n}\,\text{TRIV} = \text{TRUE}$—must also satisfy the (dynamic conjunction of the restriction and the) nuclear scope, namely, $(P\,n;\,Q\,n)\,i^{x/n}\,j^{y/n}\,\text{TRIV}$. If this test is passed, the original context is passed on to the global continuation without update.[1]

Then (4a) will evaluate to true just in case the set of boys is a subset of the set of people who entered.

The indefinite article provides a potentially distinct object for each context stack:

(5) a. A^0 boy sat.
b. $[\![a^n]\!] = \lambda PQij\kappa\,.\,\exists x, y\,.\,(P\,n;\,Q\,n)\,i^{x/n}\,j^{y/n}\,\kappa$

After update with (5a), the context will contain stacks whose first elements are boys who sat. These boys will be available for downstream anaphora.

[1] The fact that the continuation sees only the original, un-updated context means that, in the terminology of Groenendijk and Stokhof (1991), this *every* is externally static. This ensures that the binding potential of the universal (and anything in its scope) ends at the edge of its nuclear scope. However, Brasoveanu (2011:130) points toward counterexamples like (i), in which universal-internal indefinites appear to bind from beyond the grave.

(i) Harvey courts a^1 woman at every^0 convention. She$_1$ always comes to the banquet with him.

Such instances of "quantificational subordination" can be handled in this framework by moving the continuation variable into the scope of the quantifier, rendering the quantifier externally dynamic — i.e., by eliminating the final conjunct ($\kappa\,i\,j$) and replacing the second occurrence of TRIV with κ.

Universal and existential quantification interact in a reasonable way. Here and in what follows we will represent quantifiers in "scope position", i.e., in the position they would have after some operation like Quantifier Raising has occurred. The reader is encouraged to supplement the grammar with whatever scope-taking mechanism he or she prefers.

(6) a. Every^0 boy recited a^1 poem.
 b. $[\![every^0\ boy]\!]\ \big(\lambda n\,.\,[\![a^1\ poem]\!]\ (\lambda m\,.\,[\![recited]\!]\ m\ n)\big)$
 c. $[\![recited]\!] = \lambda mnij\kappa\,.\,\langle i_n, i_m\rangle, \langle j_n, j_m\rangle \in \textbf{recite} \wedge \kappa\ i\ j$
 d. $[\![poem]\!] = \lambda nij\kappa\,.\,i_n, j_n \in \textbf{poem} \wedge \kappa\ i\ j$

After scoping the two generalized quantifiers, the logical form for (6a) will be as in (6b). Here, $[\![a^1\ poem]\!]$ updates its input stacks by inserting a poem in their 1st positions, and then anticipates a predicate, which it will direct toward these positions.

$$\begin{aligned} &\lambda Qij\kappa\,.\,\exists z, z'\,.\,\big([\![poem]\!]\ 1;\ Q\ 1\big)\ i^{z/1}\ j^{z'/1}\ \kappa \\ =\ &\lambda Qij\kappa\,.\,\exists z, z'\,.\,i_1^{z/1}, j_1^{z'/1} \in \textbf{poem} \wedge Q\ 1\ i^{z/1}\ j^{z'/1}\ \kappa \end{aligned}$$

To make things a little easier to process, notice that $i_1^{z/1}$ denotes the object in the 1st position of the stack that results from inserting z into the 1st position of i. Clearly this is just z. Similarly, $j_1^{z'/1}$ is just z'. Bearing this in mind, after feeding the VP in for Q above and reducing a bit, the full nuclear scope of the universal is equivalent to

$$\lambda nij\kappa\,.\,\exists z, z'\,.\,z, z' \in \textbf{poem} \wedge \left\langle i_n^{z/1}, z\right\rangle, \left\langle j_n^{z'/1}, z'\right\rangle \in \textbf{recite} \wedge \kappa\ i^{z/1}\ j^{z'/1}.$$

Finally, factoring this property in as an argument to the distributive DP, and further simplifying along the lines above, leaves us with the following denotation for the sentence (the repetition of the restrictor in the consequent is omitted here, as it is vacuous when the restrictor does not itself introduce new discourse referents).

$$\lambda ij\kappa\,.\,\big(\forall x, y, x \neq y\,.\,x, y \in \textbf{boy} \rightarrow \exists z, z'\,.\,z, z' \in \textbf{poem} \wedge \langle x, z\rangle, \langle y, z'\rangle \in \textbf{recite}\big) \wedge \kappa\ i\ j$$

If the proposition is fed a pair of input stacks and a trivial continuation, it returns a truth value: true if for every pair of boys x and y, there are a pair of poems z and z' such that x recited z and y recited z'. The whole update process is schematically represented below, assuming the initial input stacks are both **ba**.

$$\begin{array}{lcccccc} & & & \overset{boy}{\frown} & & \overset{poem}{\frown} & & \\ i: & \textbf{b a} & \xrightarrow{every^0\ boy} & \textbf{x b a} & \xrightarrow{a^1\ poem} & \textbf{x z}_1\ \textbf{b a} & \longrightarrow & \textbf{b a} \\ j: & \textbf{b a} & & \textbf{y b a} & & \underbrace{\textbf{y z}_2}_{recite}\ \textbf{b a} & & \textbf{b a} \end{array}$$

With the distributive machinery set up, adding a lexical entry for singular *different* is straightforward.[2]

(7) a. Every^0 boy recited a^1 different poem.
b. $[\![every^0\ boy]\!]\ \big(\lambda n\,.\,[\![a^1\ different\ poem]\!]\ (\lambda m\,.\,[\![recited]\!]\ m\ n)\big)$
c. $[\![different]\!] = \lambda P m\,.\,P\ m;\ (\lambda i j \kappa\,.\,i_m \neq j_m\ \wedge\ \kappa\ i\ j)$

Now, $[\![a^1\ different\ poem]\!]$ reduces to

$$\lambda Q i j k\,.\,\exists z, z'\,.\,\big([\![poem]\!]\ 1;\ (\lambda i j k\,.\,i_1 \neq j_1\ \wedge\ \kappa\ i\ j);\ Q\ 1\big)\ i^{z/1}\ j^{z'/1}\ \kappa$$
$$=\ \lambda Q i j k\,.\,\exists z, z'\,.\,i_1^{x/1}, j_1^{y/1} \in \mathbf{poem} \wedge \boxed{i_1^{z/1} \neq j_1^{z'/1}} \wedge Q\ 1\ i^{z/1}\ j^{z'/1}\ \kappa.$$

The only difference between this function and the function defined by $[\![a^1 different\ poem]\!]$ is the boxed conjunct $i_1^{z/1} \neq j_1^{z'/1}$. Again, since $i_1^{z/1} = z$ and $j_1^{z'/1} = z'$, this requires that the poems z and z' introduced by the indefinite DP be distinct. The rest is exactly as before. We end up with a denotation equivalent to

$$\lambda i j \kappa\,.\,\big(\forall x, y, x \neq y\,.\,x, y \in \mathbf{boy} \rightarrow \exists z, z'\,.\,z, z' \in \mathbf{poem} \wedge\ z \neq z'\ \wedge \langle x, z\rangle, \langle y, z'\rangle \in \mathbf{recite}\big) \wedge \kappa\ i\ j.$$

2 An ambiguity: singular *different* can choose among multiple distributive antecedents

The fragment in Brasoveanu (2011) allows only one distributive operator at a time to control the extra information channel. The distributivity operator that is in control must always be the most local one (local in terms of narrowest scope). This is too restrictive, as the following examples show:

(8) a. Every boy gave every girl a different poem.
b. Every boy gave every girl he liked a different poem.
c. Every boy said [every girl read a different poem].
d. Every boy said [every girl read a different poem from a different book].

The potential for inverse scope provides some wiggle room. Both the fragment developed above and the one from Brasoveanu will generate two distinct readings for (8a), depending on whether *every boy* takes wide scope—in which case no boy gives

[2] In Brasoveanu (2011), *different* takes two indices: one that matches the index on the indefinite determiner it occurs immediately under, and another giving an offset value to use for finding the object to use for comparison. Neither of these indices are necessary here: the index of the indefinite is already available as the first argument to the nominal modified by *different*; and because this implementation does not make use of Brasoveanu's concatenation operator, the object to use for comparison will simply be found in the same position of the second context stack. In the generalized fragment of the next section, we will need to add a parameter distinct from any of the ones in Brasoveanu's account in order to disambiguate the antecedent of *different* for examples like (2a).

the same poem to multiple girls—or whether *every girl* does—in which case no girl receives the same poem from multiple boys. But native speakers report that (8b) shows the same ambiguity, even though the pronoun in *every girl he liked* forces the subject DP to take wider scope. Likewise, (8c) is ambiguous despite the fact that *every* DPs are generally trapped inside of tensed clauses (the extent of the embedded tensed clause is marked here with brackets). Finally, despite the fact that (8d) can have only one scoping of the universal quantifiers, it has four distinct interpretations, depending on which distributive operator each of the *different*s associates with.

The reason that Brasoveanu's implementation of the AwD strategy predicts only a single reading (per scoping) is because it allows for only a single extra information channel. So if there are two distributivity operators, in order for the operator with narrower scope to take control over the context, it must discard the information placed there by the first distributivity operator. Thus the distributivity operator with the narrowest scope will always be the only available antecedent for singular *different*.

But even once we've fixed the relative scope of the universals, it might be possible to generate the two readings of the sentences in (8a–c) by adjusting the scope of the *different* indefinite. For example, the logical form in (9) derives association-with-boys truth conditions for (8a), according to which no girl receives the same poem from multiple boys.

(9) $[\![\textit{every boy}]\!] \left(\lambda x\,.\,[\![\textit{a different poem}]\!]\,(\lambda z\,.\,[\![\textit{every girl}]\!]\,(\lambda y\,.\,[\![\textit{gave}]\!]\,z\;y\;x))\right)$

As long as *different* is not in the scope of *every girl*, there is no obstacle to taking the distributivity introduced by *every boy* as antecedent, as desired.

However, just as we fixed the scopal ordering of the universals, we can fix the scope of the indefinite, and the potential for long-distance association persists.

(10) Every$_1$ photographer claimed that [each$_2$ woman] preferred [a different$_1$ picture of herself$_2$].

In order for *each woman* to bind the anaphor *herself*, it must outscope the pronoun. Yet it remains possible for *different* to take the higher quantifier as its antecedent. The relevant reading is one that would be satisfied if each photographer claimed that each woman preferred the photograph of herself that that photographer had taken. Note that reconstruction is not relevant here: even if it were possible to reconstruct a portion of a raised indefinite, the truth conditions would then entail that each photograph was a photograph of every woman, which is not required on the relevant reading.

Just to be safe, we can use bound pronouns to impose a specific scopal order over all three quantifiers:

(11) a. Each$_1$ traffic engineer insisted that [every$_2$ intersection she$_1$ controlled] ought to have [a different$_1$ speed at which its$_2$ lights changed].
b. Each$_1$ professor asked [every$_2$ student in her$_1$ class] to present [a different$_1$ paper by his$_2$ favorite author].

If the pronouns are bound as indicated, in (11a) *each traffic engineer* must take scope over the DP *every intersection she controlled*, and the indefinite *a different speed at which its lights changed* must likewise be in the scope of the DP introduced by *every intersection*. Then since (11a) has an interpretation that entails that no two traffic engineers insist on the same speed, it must be possible for *different* to be in the scope of both distributive operators and yet still take the higher operator as its antecedent.

A second argument along similar lines comes from inverse linking:

(12) a. Each polling company interviewed [a different person from every city].
b. Every unrelated language has [a different morpheme for marking each case].

It is generally accepted (see, e.g., the discussion in Heim and Kratzer 1998, p. 233) that in cases of inverse linking, quantifiers external to the inversely linked DP cannot intervene in scope between the host and its linker. That is, on any reading of the sentences in (12) in which the embedded universal takes scope over the indefinite that contains it, the subject universal cannot take scope between the embedded universal and the indefinite. Thus since (12a) has a reading on which it entails that no two polling companies interviewed the same person, it must be possible for *different* to take *each company* as its controlling distributor despite also being in the scope of *every city*.

In view of these arguments, we will assume that it is possible for singular *different* to take a non-local distributive operator as antecedent. In order to generalize the fragment to handle this kind of ambiguity, instead of a pair of context stacks, we need a list of stacks, a list whose length is unbounded.

Types		Variables	Examples
Stacklist:	List of Stacks	c	[**ab**], [**ab**, **aac**], ...
Continuation:	Stacklist → Bool	κ	TRIV $\equiv \lambda c\,.\,$TRUE
Proposition:	Stacklist → Continuation → Bool	p, q	

As shown above, this simplifies the types of continuations and propositions, which in turn simplifies the lexical entries throughout the fragment. Where before a lexical item might have manipulated two stacks, e.g., by checking both stacks to see whether their nth element was a boy, now lexical items will act on the stacklist universally, checking for every stack (however many that may be) to see whether the nth element is a boy.

(13) a. $p; q \equiv \lambda c\kappa\,.\,p\,c\,(\lambda c'\,.\,q\,c'\,\kappa)$
b. $[\![boy]\!] = \lambda nc\kappa\,.\,(\forall i \in c\,.\,i_n \in \mathbf{boy}) \wedge \kappa\,c$
c. $[\![girl]\!] = \lambda nc\kappa\,.\,(\forall i \in c\,.\,i_n \in \mathbf{girl}) \wedge \kappa\,c$
d. $[\![poem]\!] = \lambda nc\kappa\,.\,(\forall i \in c\,.\,i_n \in \mathbf{poem}) \wedge \kappa\,c$
e. $[\![gave]\!] = \lambda mnlc\kappa\,.\,(\forall i \in c\,.\,\langle i_l, i_m, i_n\rangle \in \mathbf{give}) \wedge \kappa\,c$
f. $[\![a^n]\!] = \lambda PQc\kappa\,.\,\exists z_0, z_1, \ldots, z_m\,.\,(P\,n;\,Q\,n)\,c'\,\kappa,$
where $m = |c| - 1$ and $c' = \left[c_0^{z_0/n}, c_1^{z_1/n}, \ldots, c_m^{z_m/n}\right]$

g. $[\![every^n]\!] \quad = \lambda P Q c \kappa\,.\,\big(\forall x, y, x \neq y\,.\,P\,n\,c'\,\text{TRIV} \rightarrow (P\,n;\,Q\,n)\,c'\,\text{TRIV}\big) \wedge \kappa\,c,$
where $m = |c| - 1$ and $c' = \Big[c_0^{x/n}, c_1^{x/n}, \ldots, c_m^{x/n}, c_0^{y/n}\Big]$

h. $[\![different^n]\!] \quad = \lambda P m\,.\,P\,m;\,\big(\lambda c \kappa\,.\,(c_0)_m \neq (c_n)_m \;\wedge\; \kappa\,c\big)$

The generalized existential in (13f) searches for individuals $z_0, \ldots, z_m$ that will play satisfactory roles in the nth positions of each stack. It is exactly like the two-stack existential in (5b), except that it will attempt to fill the nth positions of as many stacks as it needs to (as many as are on the stacklist at that moment that it is evaluated). The generalized universal is only slightly more complicated than it was above. Most importantly, it tacks one additional stack onto the end of the stacklist, which is an exact copy of the first stack on the list, except for the nth coordinate. In this special position, the universal introduces two individuals x and y, one in the top stack and one in the bottom. This implements Brasoveanu's notion that distributive quantifiers give rise to dual quantification over pairs of objects. In all of the intermediate stacks (if there are any), x is inserted at index n. This is easiest to picture if we represent the stacklist as a matrix. Then "tests" of the nth position in each stack (e.g. *boy*) become tests on the nth column, and actions on the nth member of each stack become actions on the nth column.

For instance, we might represent the updates of the existential and universal determiners by the diagrams in (14a,c) and (14b,d) respectively, which provide representative examples of the effect these determiners have on contexts containing one stack (14a,b) or multiple stacks (14c,d). The indefinite is satisfied if there are *any* individuals $z_0, \ldots, z_m$ that meet the conditions of the sentence. The universal is satisfied if every column of individuals $x, \ldots, x, y$ meets the relevant conditions.

(14)

a. $[\mathbf{a}\;\;\mathbf{c}] \xrightarrow{a^1\;poem} [\mathbf{a}\;\;\overbrace{\mathbf{z}}^{poem}\;\;\mathbf{c}]$ b. $[\mathbf{a}\;\;\mathbf{c}] \xrightarrow{every^1\;boy} \begin{bmatrix} \mathbf{a} & \overbrace{\mathbf{x}}^{boy} & \mathbf{c} \\ \mathbf{a} & \mathbf{y} & \mathbf{c} \end{bmatrix}$

c. $\begin{bmatrix} \mathbf{a} & \mathbf{c} \\ \mathbf{b} & \mathbf{c} \end{bmatrix} \xrightarrow{a^1\;poem} \begin{bmatrix} \mathbf{a} & \overbrace{\mathbf{z}_0}^{poem} & \mathbf{c} \\ \mathbf{b} & \mathbf{z}_1 & \mathbf{c} \end{bmatrix}$ d. $\begin{bmatrix} \mathbf{a} & \mathbf{c} \\ \mathbf{b} & \mathbf{c} \end{bmatrix} \xrightarrow{every^1\;boy} \begin{bmatrix} \mathbf{a} & \overbrace{\mathbf{x}}^{boy} & \mathbf{c} \\ \mathbf{b} & \mathbf{x} & \mathbf{c} \\ \mathbf{a} & \mathbf{y} & \mathbf{c} \end{bmatrix}$

We can now account for the ambiguity in (8).

(15) a. $[\![every^0\;boy]\!]\Big(\lambda x\,.\,[\![every^1\;girl]\!]\big(\lambda z\,.\,[\![a^2\;different^2\;poem]\!]\,(\lambda y\,.\,[\![gave]\!]\,z\,y\,x)\big)\Big)$

b. $[\![every^0\;boy]\!]\Big(\lambda x\,.\,[\![every^1\;girl]\!]\big(\lambda z\,.\,[\![a^2\;different^1\;poem]\!]\,(\lambda y\,.\,[\![gave]\!]\,z\,y\,x)\big)\Big)$

The only difference is the choice of stack parameter indicated as a superscript on *different*. The goal is to choose the stack in the context list that differs from the top one only in its choice of the relevant distributed variable. Consider (15a). If the input context to this sentence is [**a**], then the first universal copies this stack and adds a single point of variation (two distinct boys) in column 0. The second universal copies the top stack again, this time with variation (two distinct girls) in column 1. Finally, the existential injects a non-deterministic column of poems, one in slot 2 of each of the three stacks. These updates are schematically represented below:

$$[\mathbf{a}] \xrightarrow{\textit{every}^0\ \textit{boy}} \begin{bmatrix} \overbrace{\mathbf{x}}^{\textit{boy}} & \mathbf{a} \\ \mathbf{y} & \mathbf{a} \end{bmatrix} \xrightarrow{\textit{every}^1\ \textit{girl}} \begin{bmatrix} \mathbf{x} & \overbrace{\mathbf{u}}^{\textit{girl}} & \mathbf{a} \\ \mathbf{y} & \mathbf{u} & \mathbf{a} \\ \mathbf{x} & \mathbf{v} & \mathbf{a} \end{bmatrix} \xrightarrow{a^2\ \textit{poem}} \underbrace{\begin{bmatrix} \mathbf{x} & \mathbf{u} & \overbrace{\mathbf{z}_0}^{\textit{poem}} & \mathbf{a} \\ \mathbf{y} & \mathbf{u} & \mathbf{z}_1 & \mathbf{a} \\ \mathbf{x} & \mathbf{v} & \mathbf{z}_2 & \mathbf{a} \end{bmatrix}}_{\textit{give}} \longrightarrow [\mathbf{a}]$$

Every nested universal adds one stack to the stacklist, so at any given point in the computation, there will be as many (additional) stacks as there are dominating distributors. Thus we can associate every non-initial stack with the distributor that introduced it. Because of the way that distributors induce variation in the stacks that they introduce, each of these duplicated stacks will differ from the topmost stack in exactly one column (ignoring for a second any indeterminacy introduced by existentials), the column corresponding to the associated distributor's index. For instance, the final stack above was created by the embedded DP *every*^{1} *girl*, and so it differs from the top stack exactly in column 1 (the column identified by the DP's index). In general, the narrowest scoping distributor will control the bottom (most recent) stack in the list; so the bottom stack will always differ from the top one in the column indicated by the narrowest universal's index. The next narrowest distributor will control the next lowest stack, and so on, so that the widest scoping distributor will control the second stack.

Thus, to "associate" with girls, *different* in (15a) is directed toward stack 2, which it compares to stack 0 with respect to the individuals in column 2 (which it picks up from its containing indefinite). The sentence is true iff for any boy x and any two girls u and v, there are two distinct poems z_0 and z_2 such that x gave z_0 to u and x gave z_2 to v. To associate with boys, *different* in (15b) checks out stack 1, which it again compares to stack 0 with respect to column 3. The sentence is true iff for any two boys x and y, and any girl u, there are two distinct poems z_0 and z_1 such that x gave z_0 to u and y gave z_1 to u.

3 Distributivity and computational complexity

Admittedly, on the compositional account just laid out, there can be a lot to keep track of. The default singleton stacklist records the sequence of potentially relevant individuals that have been introduced into the discourse. Then, according to the story we have told, every distributor adds another full copy of that list to the semantic memory.

Yet this is still at least a dimension less than what we find in Brasoveanu (2011), where contexts are not modeled by stacks, as here, but by sets of stacks, dubbed 'information states'. To be sure, the info-state analysis might just as well be generalized along the lines proposed here, according to which propositions are evaluated in multi-part contexts rather than merely two-part contexts. Elsewhere, Brasoveanu (2008) has argued for the value of information states in modeling the updates of singular, plural, and quantificational DPs all at once. However, in this section we will suggest that full information state matrices are not needed to account for internal *different*.

Brasoveanu (2011, pp. 149–151) shows how his info-state analysis can be extended to account for plural *different*, which in contrast to singular *different*, is licensed by various sorts of plurals, as in (16).

(16) The boys read different books.

On Brasoveanu's analysis, singular internal *different* and plural internal *different* are built from the same pieces. Universals like *every book* and definite plurals like *the books* both introduce what Brasoveanu calls "discourse-level plurality" (roughly, an info-state column full of books). *Different* always takes two indices that tell it where to look for items to compare. The only difference between sentences with singular *different* and those with plural *different* is that in the former, the universal determiner introduces distributivity over the plurality it has built ("the boys each"), whereas in the latter, *different* packs its own distributive punch, picking up where the plural left off ("the boys …each").

To extend the stack-based analysis here to handle sentences like (16), we would need to incorporate general mechanisms for representing pluralities. One way to do this without retreating to full matrix-sized contexts would be to supplement the domain with Link-style sums or sets, and let plural variables range over those collectives. Then, as in Brasoveanu, plural *different* would need to introduce its own distributivity to break those sums apart for the purposes of comparison. But we would lose any semblance of symmetry between the two *different*s. This is because universals in our fragment have nothing to do with pluralities; they are essentially true old-fashioned universal quantifiers (though now over two variables instead of just one). And so the *different* that associates with pluralities needs to interact with the sum-individual portion of the grammar, not the context-duplicating distributive machinery. On this line of approach, plural *different* introduces a pair-based version of the kind of covert distributivity that is often assumed to operate on sums, as in "The boys met and had an espresso", rather than the kind of genuine distributivity introduced by universal quantifiers. Thus we collapse Brasoveanu's matrices into vectors by compressing the columns (discourse pluralities) into sums (ontological pluralities).

In fact, it is possible to provide a compositional account that does not require adding any special contextual information to the context: the parasitic scope analysis in Barker (2007) generates the full pattern of readings in (8)–(12) without appeal to contextual information in any form. According to Barker, adjectives like *different* are scope-taking elements in their own right.

(17) a. Every boy read a different poem.
b. $[\![\textit{every boy}]\!]\ (\lambda x\,.\,x$ read a different poem$)$
c. $[\![\textit{every boy}]\!]\ \big([\![\textit{different}]\!]\ (\lambda f\,\lambda x\,.\,x$ read a f poem$)\big)$

First, the licensing DP *every boy* takes scope, as in (17b). Then *different* takes scope in between *every boy* and its nuclear scope, as shown in (17c). The ambiguity in (18a) amounts to a run-of-the-mill scope ambiguity.

(18) a. Every boy gave every girl a different poem.
b. $[\![\textit{every boy}]\!] \Big(\lambda x \,.\, [\![\textit{every girl}]\!] \,(\lambda y \,.\, x \text{ gave } y \text{ a different poem})\Big)$
c. $[\![\textit{every boy}]\!] \Big(\lambda x \,.\, [\![\textit{every girl}]\!] \,([\![\textit{different}]\!] \,(\lambda f \lambda y \,.\, x \text{ gave } y \text{ a } f \text{ poem}))\Big)$
d. $[\![\textit{every boy}]\!] \Big([\![\textit{different}]\!] \,(\lambda f \lambda x \,.\, [\![\textit{every girl}]\!] \,(\lambda y \,.\, x \text{ gave } y \text{ a } f \text{ poem}))\Big)$

Given a particular scoping of (18a), say, linear scope as in (18b), *different* is free to scope just under the narrower-scope quantifier as in (18c), taking *every girl* as its antecedent; or just under the wider-scope quantifier as in (18d), taking *every boy* as its antecedent.

We took pains above to show that scope ambiguity alone was not enough to allow the Brasoveanu (2011) fragment to account for the full range of meanings without modification. The reason scope ambiguity suffices for the parasitic scope approach is that there are more scope-taking elements involved: not only the quantifiers, but the adjectives *same* and *different* as well.

Although this accounts for the main ambiguity discussed here in a straightforward way, it is worth noting that the parasitic scope approach has nothing to say about the difference between singular and plural *different*, nor the similarity between internal and external *different* (these issues are discussed in the next section).

4 A single lexical entry?

We can now use the differences between the fragment in Brasoveanu (2011) and the analysis here to gain a deeper understanding of how AwD works in general, independently of any specific implementation.

Brasoveanu shows that allowing distributivity operators to spread information across context elements can lighten the compositional burden on adjectives like *same* and *different*. It also paves the way for a unified lexical entry for *different* that arguably captures the similarity in meaning between internal and external readings.

(19) a. Each boy read a different book.
b. Each boy read a different book than all of the other boys read. [internal]
c. Each boy read a book different from that book. [external]

The sentence in (19a) is ambiguous between the paraphrases in (19b) and (19c). Brasoveanu suggests that one advantage of providing a single lexical entry is that it explains why languages that allow internal readings generally also allow the same lexical item to participate in external readings.

We can offer something like a unified lexical entry here as well, if we relativize *different* to a pair of integer coordinates.

(20) internal-*different*^{n}: $\lambda P m \,.\, P\, m; (\lambda c \kappa \,.\, (c_0)_m \neq (c_n)_m \;\wedge\; \kappa\, c)$ (= (13h))

(21) external-*different*^{l}: $\lambda P m \,.\, P\, m;\; \underline{P\, l}; (\lambda c \kappa \,.\, (c_0)_m \neq (c_0)_l \;\wedge\; \kappa\, c)$

(22) unified-*different*^{n,l}: $\lambda P m \,.\, P\, m;\; \underline{P\, l};$
$(\lambda c \kappa \,.\, (c_0)_m \neq (c_n)_l \;\wedge\; \kappa\, c)$, where $n = 0$
or $l = m$

In the unified entry, the first integer n says which stack in the stacklist to find the comparison object in, and the second integer l says which column in the selected stack to find the comparison object in. In order to get the internal reading described above, choose $l = m$. In order to get the external reading, choose $n = 0$. Also, as Brasoveanu notes, we must add a presupposition (distinguished here from ordinary truth conditions by underlining) to the non-internal reading that guarantees the comparison object is a member of the category given by the complement of *different*. That is, if something is a *different poem*, then the thing it is different from must also be a poem.[3]

Is this really a unified entry? It would be more fair to say that the entry for the internal use bears a close resemblance to the external use, but they are in fact slightly different. Internal *different* compares items along a horizontal axis; it searches backwards through the discourse, within a particular context. External *different*, however, looks vertically for its comparates, across alternative contexts at a particular moment. Similarly, Brasoveanu's external *different* locates its antecedent in its own information state, while thanks to a special concatenation operator, internal *different* manages to see into the alternative state it is paired with.

Perhaps, though, this close but imperfect degree of similarity is the right result. It certainly makes it natural for an internal use to be generalized to an external use, without making it inevitable. Just as there are adjectives of comparison that have an external use but not internal use, as Brasoveanu notes (such as *other*, as in *John and Bill read the other book*, which cannot receive an internal reading), there may be adjectives that can receive internal readings but not external readings, such as *mutually incompatible* or *pairwise disjoint*.

Even in Brasoveanu (2011), as has already been mentioned, when *different* and *same* compare parts of a plurality, they are outfitted with their own distributive operators. But when plural *different* is bound to a sentence-external antecedent, it does not distribute over the parts of the plurality. Thus while the AwD approach succeeds in unifying internal singular *different* and external singular *different*, it makes no attempt to unify internal plural *different* and external plural *different*.

[3] Note that calling a reading 'external' is inaccurate if external means that the comparison referent comes from outside of the clause containing *different*.

(i) Every^0 boy read a^1 book that his teacher assigned to him and a different^{0,1} book that he chose himself.

Here, *different* is in the scope of a distributive operator, and the identity of the book chosen by the teacher might differ for each boy. Yet *different* can select that book, guaranteeing that each boy read two distinct books (but not guaranteeing that different boys read different books).

Finally, in addition, every account is in need of a separate, non-anaphoric lexical entry for both *same* and *different* to handle uses in which they appear with an overt relative clause complement, as in *a different book from the one I read yesterday*. In light of all this, we should be satisfied with an analysis that emphasizes the similarities between internal and external uses of these adjectives, without trying to make one use a subtype of the other.

5 Guaranteeing parallel properties of the stacks in the stacklist

In the implementation in Brasoveanu (2011), ordinary predicates like *entered* consider only the first element in the context, and ignore the remaining information. Here is how Brasoveanu puts it (p. 126):

> Furthermore, this additional information [the secondary context] is usually not accessed, even when it is available in the scope of distributive quantification. Pretty much all the updates, including the ones contributed by indefinites, pronouns, lexical relations, etc., target the left member of any input pair of info states. With one exception: items like *different* that can have sentence-internal readings.

Despite the impression given by this (accurate) description of the Brasoveanu (2011) fragment, it is crucially important to guarantee that all of the properties and relations that hold in the primary context also hold in the secondary context.

(23) Every boy recited a different poem.

That is, in (23), it is necessary to make sure that all of the objects that will eventually be compared are poems, and that they stand in the appropriate recitation relations to the appropriate boys, both in the primary context and in the secondary context. For Brasoveanu, this is ensured by the definition of the **dist** operator. In the final clause of the definition, we have that $\mathbf{dist}_{u_0}\, D\,\langle I, K\rangle\,\langle J, K'\rangle$ holds given an update function D only if for all $x \neq x'$, $D\,\langle I_{u_0=x}, J_{u_0=x'}\rangle\,\langle J_{u_0=x}, J_{u_0=x'}\rangle$. But since $x \neq x'$ entails that $x' \neq x$, we must also have $D\,\langle I_{u_0=x'}, J_{u_0=x}\rangle\,\langle J_{u_0=x'}, J_{u_0=x}\rangle$. (Here, an update D relates an input pairing of a primary context with a secondary context with an output pairing of an updated primary context along with an updated secondary context. See Brasoveanu (2011) for the details of the '$I_{u_0=x}$' notation, which restricts an information state to those stacks that have x in the 0th column.) Focusing on the primary (i.e., the leftmost) elements of these input and output pairs, the only way that $I_{u_0=x'}$ and $J_{u_0=x'}$ can be related is if $J_{u_0=x'}$ has boys and poems in the relevant columns such that each boy recited his corresponding poem. This is what guarantees that the secondary context satisfies all of the properties and relations that the primary context does.

In the implementation here, this parallelism in the non-primary context stacks is accomplished in a less elegant, but more transparent way, by simply requiring each predicate to impose its requirements on all context stacks equally. One advantage of the more straightforward technique is that it is easier to compute the update effect for concrete examples. In the Brasoveanu (2011) fragment, the obvious computational strategy is to generate all possible pairs of information states of the appropriate stack length, and then check which pairs satisfy the requirements imposed by the content of the distributive predicate; but even for toy models, this quickly becomes computationally intractable. In the fragment here, computing the set of output stacks for each input stack is straightforward and deterministic.

6 Conclusions

Association with distributivity is a viable, fully compositional account of the truth conditions of adjectives of comparison. We have discussed in detail here only singular *different*, but Brasoveanu (2011) shows how to extend the analysis to at least plural *different* and a variety of uses of *same*.

We developed a fragment building on de Groote's (2006) continuation-based dynamic semantics. In addition to providing a second implementation of AwD for comparison with the one in Brasoveanu, the application to AwD illustrates the elegance, flexibility, and utility of de Groote's technique.

On the AwD strategy, the presence of distributive operators requires at least doubling the amount of discourse information tracked by the compositional semantics. On the one hand, the formal analysis here shows that it is not necessary to track full information states (sets of stacks), as in Brasoveanu (2011), since tracking simple stacks will suffice. On the other hand, we have argued that in the general case, the number of stacks (or contexts, however they are implemented) should be equal to the number of nested distributive operators. This result depends on cases in which *different* needs to take a non-local distributive operator as its antecedent.

One of the main goals of the discussion in Brasoveanu (2011) is to arrive at a unified account of internal and external uses of at least singular *different*. We have suggested that the AwD strategy does not lead to a fully unified lexical entry, though it does provide a satisfying and appropriate account of the similarities across the two kinds of uses.

References

Barker, C. (2007). Parasitic scope. *Linguistics and Philosophy, 30*, 407–444.

Brasoveanu, A. (2008). Donkey pluralities: Plural information states versus non-atomic individuals. *Linguistics and Philosophy, 31*(2), 129–209.

Brasoveanu, A. (2011). Sentence-internal different as quantifier-internal anaphora. *Linguistics and Philosophy, 34*, 93–168.

de Groote, P. (2006). Towards a Montagovian account of dynamics. In *Proceedings of semantics and linguistic theory XVI*.

Groenendijk, J., & Stokhof, M. (1991). Dynamic predicate logic. *Linguistics and Philosophy, 14*, 39–100.

Heim, I., & Kratzer, A. (1998). *Semantics in generative grammar*. Oxford: Blackwell.