1 P&F, the basic idea

- Sentences are sensitive to *context-intervals*

(1) Mary kissed John
$\lambda i\,.\,\exists e\,.\,\mathsf{kiss}\,\mathsf{m}\,\mathsf{j}\,e \wedge \tau\,e \subseteq i$

- Temporal prepositional phrases (tPPs) are also context-iterval sensitive

(2) Mary kissed John during every meeting
$\lambda i\,.\,\forall e\,.\,\mathsf{meet}\,e \wedge \tau\,e \subseteq i \Rightarrow \exists e'\,.\,\mathsf{kiss}\,\mathsf{m}\,\mathsf{j}\,e' \wedge \tau\,e' \subseteq \tau\,e$

- And the interval dependencies in these tPPs tend to *cascade*, according to the order in which they scope

(3) Mary kissed John during every meeting one Monday
$\lambda i\,.\,\exists e\,.\,\mathsf{mond}\,e \wedge \tau\,e \subseteq i \wedge \forall e'\,.\,\mathsf{meet}\,e' \wedge \tau\,e' \subseteq \tau\,e \Rightarrow \exists e''\,.\,\mathsf{kiss}\,\mathsf{m}\,\mathsf{j}\,e'' \wedge \tau\,e'' \subseteq \tau\,e'$

- Sentences and nominals both expose an event variable that quantifiers (and closure operators) can bind:
  - some meeting ~ [(at some point) Mary arrived]
  - the meeting ~ before [(the time that) Mary arrived]
  - every meeting ~ when [(every time that) Mary arrived]

  Determined sentences and DPs both denote generalized quantifiers over intervals (this is how they're going to pass interval-restrictions down):
  - $[\![\text{some meeting}]\!] \approx \lambda P i\,.\,\exists e\,.\,\mathsf{meet}\,e \wedge \tau\,e \subseteq i \wedge P\,(\tau\,e)$
  - $[\![\text{Mary arrived}]\!] \approx \lambda P i\,.\,\exists e\,.\,\mathsf{arrive}\,e \wedge \tau\,e \subseteq i \wedge P\,(\tau\,e)$

2 P&F, some details

2.1 P&F combinators

- Pseudoapplication 1:
$Q \star f = \lambda P i\,.\,Q\,(\lambda x\,.\,f\,x\,i)\,P$
- Pseudoapplication 2:
$\psi \bullet \phi = \psi \circ \phi$
- Finalization:
$m^{\downarrow} = m\,(\lambda i\,.\,\mathsf{T})$

2.2 P&F Lexicon

Phrase	*Type*	*Denotation*
meeting	eit	$\lambda x_0 i\,.\,\mathsf{meet}\,x_0 \wedge \tau\,x_0 \subseteq i$
Monday	eit	$\lambda x_0 i\,.\,\mathsf{monday}\,x_0 \wedge \tau\,x_0 \subseteq i$
every	$(et)(et)t$	$\lambda Q P\,.\,\forall x\,.\,Q\,x \Rightarrow P\,x$
a/one	$(et)(et)t$	$\lambda Q P\,.\,\exists x\,.\,Q\,x \wedge P\,x$
during/on	$((et)it)(et)it$	$\lambda \mathcal{P} P i\,.\,\mathcal{P}\,(\lambda y\,.\,P\,(\tau\,y))\,i$
the	$(et)(et)t$	$\lambda Q P\,.\,\iota x : Q\,x\,.\,P\,x$
Mary called John	eit	$\lambda x_0 i\,.\,\mathsf{call}\,\mathsf{j}\,\mathsf{m}\,x_0 \wedge \tau\,x_0 \subseteq i$

2.3 P&F Examples

$$\begin{aligned}[\![\text{a Monday}]\!] &= [\![\text{a}]\!] \star [\![\text{Monday}]\!] \\ &= \lambda P i\,.\,[\![\text{a}]\!]\,(\lambda x\,.\,[\![\text{Monday}]\!]\,x\,i)\,P \\ &= \lambda P i\,.\,\exists x_0\,.\,\mathsf{monday}\,x_0 \wedge \tau\,x_0 \subseteq i \wedge P\,x_0\end{aligned}$$

$$\begin{aligned}[\![\text{on a Monday}]\!] &= [\![\text{on}]\!]\,[\![\text{a Monday}]\!] \\ &= \lambda P i\,.\,\exists x_0\,.\,\mathsf{mond}\,x_0 \wedge \tau\,x_0 \subseteq i \wedge P\,(\tau\,x_0)\end{aligned}$$

$$\begin{aligned}[\![\text{every meeting}]\!] &= [\![\text{every}]\!] \star [\![\text{meeting}]\!] \\ &= \lambda P i\,.\,[\![\text{every}]\!]\,(\lambda x\,.\,[\![\text{meeting}]\!]\,x\,i)\,P \\ &= \lambda P i\,.\,\forall x_0\,.\,\mathsf{meet}\,x_0 \wedge \tau\,x_0 \subseteq i \Rightarrow P\,x_0\end{aligned}$$

$$\begin{aligned}[\![\text{during every meeting}]\!] &= [\![\text{during}]\!]\,[\![\text{every meeting}]\!] \\ &= \lambda P i\,.\,\forall x_0\,.\,\mathsf{meet}\,x_0 \wedge \tau\,x_0 \subseteq i \Rightarrow P\,(\tau\,x_0)\end{aligned}$$

2.3.1 Nominal Modification Before Determination

(4) every [meeting on a Monday]

$$\begin{aligned}&[\![\text{meeting on a Monday}]\!] \\ &= [\![\text{on a Monday}]\!] \bullet [\![\text{meeting}]\!] \\ &= \lambda x_0\,.\,[\![\text{on a Monday}]\!]\,(\lambda i\,.\,\mathsf{meet}\,x_0 \wedge \tau\,x_0 \subseteq i) \\ &= \lambda x_0 i\,.\,\exists x_1\,.\,\mathsf{mond}\,x_1 \wedge \tau\,x_1 \subseteq i \wedge \mathsf{meet}\,x_0 \wedge \tau\,x_0 \subseteq \tau\,x_1\end{aligned}$$

$$\begin{aligned}&[\![\text{every meeting on a Monday}]\!] \\ &= [\![\text{every}]\!] \star [\![\text{meeting on a Monday}]\!] \\ &= \lambda P i\,.\,\forall x_0\,.\,\exists x_1\,.\,\mathsf{mond}\,x_1 \wedge \tau\,x_1 \subseteq i \wedge \mathsf{meet}\,x_0 \wedge \tau\,x_0 \subseteq \tau\,x_1 \Rightarrow P\,x_0\end{aligned}$$

After Determination

(5) [one day] [of every week]

$[\![\text{of every week}]\!] = \lambda P i\,.\,\forall x_0\,.\,\mathsf{week}\,x_0 \wedge \tau\,x_0 \subseteq i \Rightarrow P\,(\tau\,x_0)$

$[\![\text{one day}]\!] = \lambda P i\,.\,\exists x_0\,.\,\mathsf{day}\,x_0 \wedge \tau\,x_0 \subseteq i \wedge P\,x_0$

$[\![\text{of every week}]\!] \bullet [\![\text{one day}]\!]$
$= \lambda P i\,.\,\forall x_0\,.\,\mathsf{week}\,x_0 \wedge \tau\,x_0 \subseteq i \Rightarrow \exists x_1\,.\,\mathsf{day}\,x_1 \wedge \tau\,x_1 \subseteq \tau\,x_0 \wedge P\,x_1$

2.3.2 Sentential Modification

Before Determination

(6) When[ever [Bill slept on a Monday]]

$[\![\text{Bill slept}]\!] = \lambda x_0 i\,.\,\mathsf{sleep}\,\mathsf{b}\,x_0 \wedge \tau\,x_0 \subseteq i$

$[\![\text{on a Monday}]\!] \bullet [\![\text{Bill slept}]\!]$
$= \lambda x_0 i\,.\,\exists x_1\,.\,\mathsf{mond}\,x_1 \wedge \tau\,x_1 \subseteq i \wedge \mathsf{sleep}\,\mathsf{b}\,x_0 \wedge \tau\,x_0 \subseteq \tau\,x_1$

$[\![\text{ever Bill slept on a Monday}]\!]$
$= [\![\text{every}]\!] \star [\![\text{Bill slept on a Monday}]\!]$
$= \lambda P i\,.\,\forall x_0\,.\,\exists x_1\,.\,\mathsf{mond}\,x_1 \wedge \tau\,x_1 \subseteq i \wedge \mathsf{sleep}\,\mathsf{b}\,x_0 \wedge \tau\,x_0 \subseteq \tau\,x_1 \Rightarrow P\,x_0$

$[\![\text{When[ever Bill slept on a Monday]}]\!]$
$= [\![\text{whenever}]\!]\,[\![\text{Bill slept on a Monday}]\!]$
$= \lambda P i\,.\,\forall x_0\,.\,\exists x_1\,.\,\mathsf{mond}\,x_1 \wedge \tau\,x_1 \subseteq i \wedge \mathsf{sleep}\,\mathsf{b}\,x_0 \wedge \tau\,x_0 \subseteq \tau\,x_1 \Rightarrow P\,(\tau\,x_0)$

After Determination

(7) Mary called John during every meeting

$\left|[\![\text{Mary called John}]\!]\right|_{[\![\text{a}]\!]} = [\![\text{a}]\!] \star [\![\text{Mary called John}]\!]$
$= \lambda P i\,.\,\exists x_0\,.\,\mathsf{call}\,\mathsf{j}\,\mathsf{m}\,x_0 \wedge \tau\,x_0 \subseteq i \wedge P\,x_0$

$[\![\text{during every meeting}]\!] \bullet [\![\text{Mary called John}]\!]$
$= \lambda P\,.\,[\![\text{during every meeting}]\!]\,([\![\text{Mary called John}]\!]\,P)$
$= \lambda P i\,.\,\forall x_0\,.\,\mathsf{meet}\,x_0 \wedge \tau\,x_0 \subseteq i \Rightarrow \exists x_1\,.\,\mathsf{call}\,\mathsf{j}\,\mathsf{m}\,x_1 \wedge \tau\,x_1 \subseteq \tau\,x_0 \wedge P\,x_1$

3 But what are those pseudoapplicators?

- ($\star$) is a special-purpose tool designed to smuggle an input interval into the restrictor of a quantifier. ($\bullet$) is a (less) special-purpose tool that effectively passes one constituent in as an argument to another.
- This all feels like a trick to effect a little bit of scope and a little bit of binding, and both of those combinators smell like continuations. In fact, if we η-expand and flip the order in which expressions take their scopes and intervals:
  $\psi \bullet \phi = \psi \circ \phi$
  $= \lambda k\,.\,\psi\,(\phi\,k)$
  $= \lambda k\,.\,\psi\,(\lambda i'\,.\,\phi\,k\,i')$
  $= \lambda k i\,.\,\psi\,(\lambda i'\,.\,\phi\,k\,i')\,i$
  [flip]
  $\lambda i k\,.\,\psi\,i\,(\lambda i'\,.\,\phi\,i'\,k)$
  We get the dynamic composition operator from de Groote 2006, "Towards a Montagovian account of dynamic semantics"

  > We now face the following question: how does one compose meanings of sentences in order to get the meaning of a discourse? Let D be a piece of discourse, and S be a sentence. The semantics of the discourse made of D and S is given by the following semantic equation:
  >
  > (3) $[\![D.S]\!] = \lambda e\phi\,.\,[\![D]\!]\,e\,(\lambda e'\,.\,[\![S]\!]\,e'\,\phi)$

- Similarly if we flip the types for the arguments of ($\star$),
  $Q \star f = \lambda i k\,.\,Q\,(\lambda x\,.\,f\,x\,i)\,k$
  $= \lambda i\,.\,Q\,(\lambda x\,.\,f\,x\,i)$
  we get Greg's Forward Permutation Composition combinator, **B**Q(**C**f), otherwise known as the bind of the continuation monad. In fact, from this perspective, ($\bullet$) is the Kliesli compositor (>=>) to $\star$.

4 P&F-dG

4.1 P&F-dG combinators

- $Q \star f = \lambda i k\,.\,Q\,(\lambda x\,.\,f\,x\,i)\,k$

- $\psi \bullet \phi = \lambda i k\,.\,\psi\,i\,(\lambda i'\,.\,\phi\,i'\,k)$

4.2 P&F-dG Lexicon

Phrase	*Type*	*Denotation*
during every meeting	$i(it)t$	$\lambda ik\,.\,\forall x_0\,.\,\mathsf{meet}\,x_0 \wedge \tau\,x_0 \subseteq i \Rightarrow k\,(\tau\,x_0)$
on one Monday	$i(it)t$	$\lambda ik\,.\,\exists x_1\,.\,\mathsf{mond}\,x_1 \wedge \tau\,x_1 \subseteq i \wedge k\,(\tau\,x_1)$
Mary called John	eit	$\lambda x_0 i\,.\,\mathsf{call\,j\,m}\,x_0 \wedge \tau\,x_0 \subseteq i$

4.3 P&F-dG Examples

$$\begin{aligned}[\![\text{(a) Mary called John}]\!] &= [\![\text{a}]\!] \star [\![\text{Mary called John}]\!]\\ &= \lambda ik\,.\,\exists x_0\,.\,\mathsf{call\,j\,m}\,x_0 \wedge \tau\,x_0 \subseteq i \wedge k\,x_0\end{aligned}$$

$$\begin{aligned}&[\![\text{during every meeting}]\!] \bullet [\![\text{Mary called John}]\!]\\ &= \lambda ik\,.\,[\![\text{during every meeting}]\!]\,i\,(\lambda i'\,.\,[\![\text{Mary called John}]\!]\,i'\,k)\\ &= \lambda ik\,.\,\forall x_0\,.\,\mathsf{meet}\,x_0 \wedge \tau\,x_0 \subseteq i \Rightarrow \exists x_1\,.\,\mathsf{call\,j\,m}\,x_1 \wedge \tau\,x_1 \subseteq \tau\,x_0 \wedge k\,x_1\end{aligned}$$

$$\begin{aligned}&[\![\text{On one Monday}]\!] \bullet [\![\text{Mary called John during every meeting}]\!]\\ &= \lambda ik\,.\,[\![\text{on a Monday}]\!]\,i\,(\lambda i'\,.\,[\![\text{Mary called John during every meeting}]\!]\,i'\,k)\\ &= \lambda ik\,.\,\exists x_2\,.\,\mathsf{mond}\,x_2 \wedge \tau\,x_2 \subseteq i \wedge\\ &\qquad \forall x_0\,.\,\mathsf{meet}\,x_0 \wedge \tau\,x_0 \subseteq \tau\,x_2 \Rightarrow \exists x_1\,.\,\mathsf{call\,j\,m}\,x_1 \wedge \tau\,x_1 \subseteq \tau\,x_0 \wedge k\,x_1\end{aligned}$$

- We could derive the other scope by composing the modifiers in the other order (same as P&F)
- But what about modifiers that attach to *undetermined* things? We've lost the ability to compose (with $\bullet$) things like [on a Monday] — type $i(it)t$ — with things like [meeting] — type eit.

5 P&F-K

- Ok, let's put the arguments back in the original order, and focus on ($\star$) instead of ($\bullet$).
- $Q \star f = \lambda ki\,.\,Q\,(\lambda x\,.\,f\,x\,i)\,k$. The reason we have to pass i down to the restrictor f is that things like meeting and arrive m expect it: $[\![\text{meeting}]\!] = \lambda xi\,.\,\mathsf{meet}\,x \wedge \tau\,x \subseteq i$. But should they?
- What if instead of passing i directly to the meeting (and every other nominal/sentence radical), we instead wrapped meeting in some sort of handler that would take care of the boilerplate temporal situating. Something like: $[\![\text{meeting}]\!] = \lambda xk\,.\,k\,(\mathsf{meet}\,x)$.
- Then when we needed to, we could drop in a closure continuation along the lines of $(\lambda fij\,.\,f\,j \wedge j \subseteq i)$ to enforce the contextual restriction. I think this is what Chris has in mind when he suggests we move from half to full continuations.
- In that case, what we need to thread down into the restrictor is not a temporal interval, but a continuation that is itself waiting for an input interval. Then all we have to do with the interval is hand it off to the quantifier, which can then as need be pass it into the continuized restrictor and/or its nuclear scope.
- In other words, interestingly, we reverse the role that k and i play in ($\star$):

  $$\begin{aligned}Q \star f &= \lambda ki\,.\,Q\,(\lambda x\,.\,f\,x\,k)\,i\\ &= \lambda k\,.\,Q\,(\lambda x\,.\,f\,x\,k)\end{aligned}$$

- And now we're back to the continuation monad bind, but for real this time. With this psuedoapplicator, we also get all the applicative goodness of the Barker-Shan towers, so I'm going to switch over.

5.1 P&F-K combinators

- $\begin{aligned}Q \star f &= \lambda ki\,.\,Q\,(\lambda x\,.\,f\,x\,k)\,i\\ &= \lambda k\,.\,Q\,(\lambda x\,.\,f\,x\,k)\end{aligned}$
- $m^{\downarrow} = m\,(\lambda fi\,.\,\{j \mid f\,j \wedge j \subseteq i\})$
- $\begin{aligned}L \setminus R &= L \star (\lambda x\,.\,R \star (\lambda f\,.\,\eta\,(f\,x)))\\ &= \lambda k\,.\,L\,(\lambda x\,.\,R\,(\lambda f\,.\,k\,(f\,x)))\end{aligned}$
- $\begin{aligned}L\,/\,R &= L \star (\lambda f\,.\,R \star (\lambda x\,.\,\eta\,(f\,x)))\\ &= \lambda k\,.\,L\,(\lambda f\,.\,R\,(\lambda x\,.\,k\,(f\,x)))\end{aligned}$

5.2 P&F-K Lexicon

Phrase	*Type* ($\mathbb{T} \equiv i\{i\}$)	*Denotation*
John	$(e\mathbb{T})\mathbb{T}$	$\lambda k\,.\,k\,\mathsf{j}$
slept	$((eit)\mathbb{T})\mathbb{T}$	$\lambda k\,.\,k\,\mathsf{sleep}$
meeting	$((eit)\mathbb{T})\mathbb{T}$	$\lambda k\,.\,k\,\mathsf{meet}$
every	$(e\mathbb{T})(e\mathbb{T})\mathbb{T}$	$\lambda cki\,.\,\{i \mid \forall x\,.\,c\,x\,i \neq \emptyset \Rightarrow \exists j \in c\,x\,i\,.\,k\,x\,j \neq \emptyset\}$
a	$(e\mathbb{T})(e\mathbb{T})\mathbb{T}$	$\lambda cki\,.\,\bigcup\{k\,x\,j \mid j \in c\,x\,i\}$
the	$(e\mathbb{T})(e\mathbb{T})\mathbb{T}$	$\lambda cki\,.\,\{i \mid \iota x\colon c\,x\,i \neq \emptyset\,.\,\exists j \in c\,x\,i\,.\,k\,x\,j \neq \emptyset\}$
during/on	$((e\alpha\alpha)\mathbb{T})\mathbb{T}$	$\lambda k\,.\,k\,(\lambda\_a\,.\,a)$

5.3 P&F-KS Examples

$\llbracket$every meeting$\rrbracket$

$$= (\llbracket\text{every}\rrbracket \backslash \llbracket\text{meeting}\rrbracket)^{\downarrow}$$

$$= \left(\frac{\llbracket\text{every}\rrbracket\,(\lambda x\,.\,[\,])}{x} \Big\backslash \frac{[\,]}{\mathsf{meet}}\right)^{\downarrow}$$

$$= \left(\frac{\llbracket\text{every}\rrbracket\,(\lambda x\,.\,[\,])}{\mathsf{meet}\;x}\right)^{\downarrow}$$

$$= \llbracket\text{every}\rrbracket\,(\lambda x i\,.\,\{j \mid \mathsf{meet}\;x\;j \wedge j \subseteq i\})$$

$$= \lambda k i\,.\,\{i \mid \forall x\,.\,\{j \mid \mathsf{meet}\;x\;j \wedge j \subseteq i\} \neq \emptyset \Rightarrow \exists j\,.\,\mathsf{meet}\;x\;j \wedge j \subseteq i \wedge k\;x\;j \neq \emptyset\}$$

$$= \frac{\lambda i\,.\,\{i \mid \forall x\,.\,\{j \mid \mathsf{meet}\;x\;j \wedge j \subseteq i\} \neq \emptyset \Rightarrow \exists j\,.\,\mathsf{meet}\;x\;j \wedge j \subseteq i \wedge [\,]\;j \neq \emptyset\}}{x}$$

$\llbracket$during every meeting$\rrbracket$

$$= \llbracket\text{during}\rrbracket \,/\, \llbracket\text{every meeting}\rrbracket$$

$$= \frac{\lambda i\,.\,\{i \mid \forall x\,.\,\{j \mid \mathsf{meet}\;x\;j \wedge j \subseteq i\} \neq \emptyset \Rightarrow \exists j\,.\,\mathsf{meet}\;x\;j \wedge j \subseteq i \wedge [\,]\;j \neq \emptyset\}}{\lambda a\,.\,a}$$

$\llbracket$John slept during every meeting$\rrbracket$

$$= (\llbracket\text{John slept}\rrbracket \backslash \llbracket\text{during every meeting}\rrbracket)^{\downarrow}$$

$$= \left(\frac{\lambda i\,.\,\{i \mid \forall x\,.\,\{j \mid \mathsf{meet}\;x\;j \wedge j \subseteq i\} \neq \emptyset \Rightarrow \exists j\,.\,\mathsf{meet}\;x\;j \wedge j \subseteq i \wedge [\,]\;j \neq \emptyset\}}{\mathsf{sleep\;j}}\right)^{\downarrow}$$

$$= \lambda i\,.\,\{i \mid \forall x\,.\,\{j \mid \mathsf{meet}\;x\;j \wedge j \subseteq i\} \neq \emptyset \Rightarrow \exists j\,.\,\mathsf{meet}\;x\;j \wedge j \subseteq i \wedge \mathsf{sleep\;j}\;j \neq \emptyset\}$$

$\llbracket$(the) Monday$\rrbracket^{\downarrow}$

$$= (\llbracket\text{the}\rrbracket \backslash \llbracket\text{Monday}\rrbracket)^{\downarrow}$$

$$= \left(\frac{\llbracket\text{the}\rrbracket\,(\lambda x\,.\,[\,])}{x} \Big\backslash \frac{[\,]}{\mathsf{mon}}\right)^{\downarrow}$$

$$= \llbracket\text{the}\rrbracket\,(\lambda x i\,.\,\{j \mid \mathsf{mon}\;x\;j \wedge j \subseteq i\})$$

$$= \frac{\lambda i\,.\,\{i \mid \iota x\colon \mathsf{mon}\;x\;i \neq \emptyset\,.\,\exists j\,.\,\mathsf{mon}\;x\;j \wedge j \subseteq i \wedge [\,]\;j \neq \emptyset\}}{x}$$

$$\llbracket\text{on Monday}\rrbracket = \llbracket\text{on}\rrbracket \,/\, \llbracket\text{(the) Monday}\rrbracket$$

$$= \frac{[\,]}{\lambda e a\,.\,a} \Big/ \dfrac{\lambda i\,.\,\{\_ \mid \iota x}{\dfrac{e\colon \mathsf{mon}\;x\;e \wedge \tau\;e \subseteq i\,.\,[\,]\,(\tau\;e) \neq \emptyset\}}{x}}$$

$$= \dfrac{\lambda i\,.\,\{\_ \mid \iota x}{\dfrac{e\colon \mathsf{mon}\;x\;e \wedge \tau\;e \subseteq i\,.\,[\,]\,(\tau\;e) \neq \emptyset\}}{\lambda a\,.\,a}}$$

$\llbracket$On Monday, John slept during every meeting$\rrbracket$

$$= (\llbracket\text{on Monday}\rrbracket \,/\, \llbracket\text{John slept during every meeting}\rrbracket)^{\downarrow}$$

$$= \left(\dfrac{\lambda i\,.\,\{\_ \mid \iota x}{\dfrac{e\colon \mathsf{mon}\;x\;e \wedge \tau\;e \subseteq i\,.\,[\,]\,(\tau\;e) \neq \emptyset\}}{\lambda a\,.\,a}} \Bigg/ \dfrac{\lambda i'\,.\,\{\_ \mid \forall x}{\dfrac{e'\,.\,\mathsf{meet}\;x\;e' \wedge \tau\;e' \subseteq i' \Rightarrow [\,]\,(\tau\;e') \neq \emptyset\}}{\mathsf{sleep\;j}}}\right)^{\downarrow}$$

$$= \left(\dfrac{\lambda i\,.\,\{\_ \mid \iota x}{\dfrac{e\colon \mathsf{mon}\;x\;e \wedge \tau\;e \subseteq i\,.\,\forall x}{\dfrac{e'\,.\,\mathsf{meet}\;x\;e' \wedge \tau\;e' \subseteq \tau\;e \Rightarrow [\,]\,(\tau\;e') \neq \emptyset\}}{\mathsf{sleep\;j}}}}\right)^{\downarrow}$$

$$= \text{TRUE iff } \iota x, e\colon \mathsf{mon}\;x\;e \wedge \tau\;e \subseteq i_{*}\,.\,\forall x', e'\,.\,\mathsf{meet}\;x'\;e' \wedge \tau\;e' \subseteq \tau\;e \Rightarrow \exists e''\,.\,\mathsf{sleep\;j}\;e$$

$\llbracket$John slept [during every meeting] [on Monday]$\rrbracket$

$$= \left(\llbracket\text{John slept during every meeting}\rrbracket^{\uparrow} \backslash \llbracket\text{on Monday}\rrbracket^{\upuparrows}\right)^{\downdownarrows}$$

$$= \left(\dfrac{[\,]}{\dfrac{\lambda i'\,.\,\{\_ \mid \forall x}{\dfrac{e'\,.\,\mathsf{meet}\;x\;e' \wedge \tau\;e' \subseteq i' \Rightarrow [\,]\,(\tau\;e') \neq \emptyset\}}{\mathsf{sleep\;j}}}} \Bigg\backslash \dfrac{\lambda i\,.\,\{\_ \mid \iota x}{\dfrac{e\colon \mathsf{mon}\;x\;e \wedge \tau\;e \subseteq i\,.\,[\,]\,(\tau\;e) \neq \emptyset\}}{\dfrac{[\,]}{\lambda a\,.\,a}}}\right)^{\downdownarrows}$$

$$= \left(\dfrac{\lambda i\,.\,\{\_ \mid \iota x}{\dfrac{e\colon \mathsf{mon}\;x\;e \wedge \tau\;e \subseteq i\,.\,[\,]\,(\tau\;e) \neq \emptyset\}}{\dfrac{\lambda i'\,.\,\{\_ \mid \forall x}{\dfrac{e'\,.\,\mathsf{meet}\;x\;e' \wedge \tau\;e' \subseteq i' \Rightarrow [\,]\,(\tau\;e') \neq \emptyset\}}{\mathsf{sleep\;j}}}}}\right)^{\downdownarrows}$$

$$= \text{TRUE iff } \iota x, e\colon \mathsf{mon}\;x\;e \wedge \tau\;e \subseteq i_{*}\,.\,\forall x', e'\,.\,\mathsf{meet}\;x'\;e' \wedge \tau\;e' \subseteq \tau\;e \Rightarrow \exists e''\,.\,\mathsf{sleep\;j}\;e$$

$$
\begin{aligned}
\llbracket \text{a Monday} \rrbracket &= (\llbracket \text{a} \rrbracket \backslash \llbracket \text{Monday} \rrbracket)^{\downarrow} \\
&= \left(\frac{\llbracket \text{a} \rrbracket\, (\lambda x .\, [\,])}{x} \;\Big\backslash\; \frac{[\,]}{\mathsf{mon}} \right)^{\downarrow} \\
&= \left(\frac{\llbracket \text{a} \rrbracket\, (\lambda x .\, [\,])}{\mathsf{mon}\, x} \right)^{\downarrow} \\
&= \llbracket \text{a} \rrbracket\, (\lambda x i .\, \{ e \mid \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \}) \\
&= \frac{\lambda i .\, \bigcup \{ [\,](\tau\, e) \mid e \in \{ e \mid \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \} \}}{x}
\end{aligned}
$$

$$
\begin{aligned}
\llbracket \text{on a Monday} \rrbracket &= \llbracket \text{on} \rrbracket \,/\, \llbracket \text{a Monday} \rrbracket \\
&= \left(\frac{[\,]}{\lambda \_ a .\, a} \;\Big/\; \frac{\lambda i .\, \bigcup \{ [\,](\tau\, e) \mid \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \}}{x} \right) \\
&= \frac{\lambda i .\, \bigcup \{ [\,](\tau\, e) \mid \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \}}{\lambda a .\, a}
\end{aligned}
$$

$$
\begin{aligned}
\llbracket \text{meeting on a Monday} \rrbracket &= \llbracket \text{meeting} \rrbracket \backslash \llbracket \text{on a Monday} \rrbracket \\
&= \frac{[\,]}{\mathsf{meet}} \;\Big\backslash\; \frac{\lambda i .\, \bigcup \{ [\,](\tau\, e) \mid \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \}}{\lambda a .\, a} \\
&= \frac{\lambda i .\, \bigcup \{ [\,](\tau\, e) \mid \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \}}{\mathsf{meet}}
\end{aligned}
$$

$$
\begin{aligned}
&\llbracket \text{every meeting on a Monday} \rrbracket \\
&= \llbracket \text{every} \rrbracket \backslash \llbracket \text{meeting on a Monday} \rrbracket \\
&= \left(\frac{\llbracket \text{every} \rrbracket\, (\lambda x' i .\, [\,]\, i)}{x'} \;\Big\backslash\; \frac{\lambda i .\, \bigcup \{ [\,](\tau\, e) \mid \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \}}{\mathsf{meet}} \right)^{\downarrow} \\
&= \left(\frac{\llbracket \text{every} \rrbracket\, (\lambda x' i .\, \bigcup \{ [\,](\tau\, e) \mid \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \})}{\mathsf{meet}\, x'} \right)^{\downarrow} \\
&= \llbracket \text{every} \rrbracket\, (\lambda x' i .\, \bigcup \{ \{ e' \mid \mathsf{meet}\, x'\, e' \wedge \tau\, e' \subseteq \tau\, e \} \mid \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \}) \\
&= \llbracket \text{every} \rrbracket\, (\lambda x' i .\, \{ e' \mid \mathsf{meet}\, x'\, e' \wedge \tau\, e' \subseteq \tau\, e \wedge \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \}) \\
&= \frac{\dfrac{\lambda i .\, \{ \_ \mid \forall x'}{e' .\, e' \in \{ e' \mid \mathsf{meet}\, x'\, e' \wedge \tau\, e' \subseteq \tau\, e \wedge \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \} \Rightarrow [\,](\tau\, e') \neq \emptyset \}}}{x'}
\end{aligned}
$$

$$
\begin{aligned}
&\llbracket \text{John slept during every [meeting on a Monday]} \rrbracket \\
&= (\llbracket \text{John slept} \rrbracket \backslash \llbracket \text{during every meeting on a Monday} \rrbracket)^{\downarrow} \\
&= \left(\frac{[\,]}{\mathsf{sleep}\, \mathsf{j}} \;\Big\backslash\; \frac{\dfrac{\lambda i .\, \{ \_ \mid \forall x'}{e' .\, e' \in \{ e' \mid \mathsf{meet}\, x'\, e' \wedge \tau\, e' \subseteq \tau\, e \wedge \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \} \Rightarrow [\,](\tau\, e') \neq \emptyset}}{\lambda a .\, a} \right. \\
&= \left(\frac{\dfrac{\lambda i .\, \{ \_ \mid \forall x'}{e' .\, e' \in \{ e' \mid \mathsf{meet}\, x'\, e' \wedge \tau\, e' \subseteq \tau\, e \wedge \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \} \Rightarrow [\,](\tau\, e') \neq \emptyset \}}}{\mathsf{sleep}\, \mathsf{j}} \right)^{\downarrow} \\
&= \mathsf{TRUE}\ \text{ iff }\ \forall x', e' .\, e' \in \{ e' \mid \mathsf{meet}\, x'\, e' \wedge \tau\, e' \subseteq \tau\, e \wedge \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \} \Rightarrow \exists e'' .\, \mathsf{sle}
\end{aligned}
$$

$$
\begin{aligned}
&\llbracket \text{John skipped every [meeting on a Monday]} \rrbracket \\
&= (\llbracket \text{John} \rrbracket \backslash \llbracket \text{skipped} \rrbracket \,/\, \llbracket \text{every meeting on a Monday} \rrbracket)^{\downarrow} \\
&= \left(\frac{[\,]}{\mathsf{j}} \;\Big\backslash\; \frac{[\,]}{\mathsf{skip}} \;\Big/\; \frac{\dfrac{\lambda i .\, \{ \_ \mid \forall x'}{e' .\, e' \in \{ e' \mid \mathsf{meet}\, x'\, e' \wedge \tau\, e' \subseteq \tau\, e \wedge \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \} \Rightarrow [\,](\tau\, e'}}{x'} \right. \\
&= \left(\frac{\dfrac{\lambda i .\, \{ \_ \mid \forall x'}{e' .\, e' \in \{ e' \mid \mathsf{meet}\, x'\, e' \wedge \tau\, e' \subseteq \tau\, e \wedge \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \} \Rightarrow [\,](\tau\, e') \neq \emptyset \}}}{\mathsf{skip}\, \mathsf{j}\, x'} \right)^{\downarrow} \\
&= \mathsf{TRUE}\ \text{ iff }\ \forall x', e' .\, e' \in \{ e' \mid \mathsf{meet}\, x'\, e' \wedge \tau\, e' \subseteq \tau\, e \wedge \mathsf{mon}\, x\, e \wedge \tau\, e \subseteq i \} \Rightarrow \exists e'' .\, \mathsf{sk}
\end{aligned}
$$

$$
\begin{aligned}
\llbracket \text{day of every week} \rrbracket &= \llbracket \text{day} \rrbracket^{\uparrow} \backslash \llbracket \text{of} \rrbracket^{\uparrow} \,/\, \llbracket \text{every week} \rrbracket^{\uparrow\uparrow} \\
&= \frac{\dfrac{[\,]}{[\,]}}{\mathsf{day}} \;\Big\backslash\; \frac{\dfrac{[\,]}{[\,]}}{\lambda \_ a .\, a} \;\Big/\; \frac{\dfrac{\dfrac{\lambda i .\, \{ \_ \mid \forall x}{e .\, \mathsf{week}\, x\, e \wedge \tau\, e \subseteq i \Rightarrow [\,](\tau\, e) \neq \emptyset \}}}{[\,]}}{x} \\
&= \frac{\dfrac{\dfrac{\lambda i .\, \{ \_ \mid \forall x}{e .\, \mathsf{week}\, x\, e \wedge \tau\, e \subseteq i \Rightarrow [\,](\tau\, e) \neq \emptyset \}}}{[\,]}}{\mathsf{day}}
\end{aligned}
$$

$$[\![\text{one day of every week}]\!] = [\![\text{one}]\!]^{\uparrow} \backslash [\![\text{day of every week}]\!]$$

$$= \begin{array}{c} [\,] \\ \hline [\![\mathrm{a}]\!]\,(\lambda x'.\,[\,]) \\ \hline x' \end{array} \Bigg\backslash \begin{array}{c} \lambda i.\,\{\_ \mid \forall x \\ \hline e.\,\mathsf{week}\,x\,e \wedge \tau\,e \subseteq i \Rightarrow [\,]\,(\tau\,e) \neq \emptyset\} \\ \hline [\,] \\ \hline \mathsf{day} \end{array}$$

$$= \begin{array}{c} \lambda i.\,\{\_ \mid \forall x \\ \hline e.\,\mathsf{week}\,x\,e \wedge \tau\,e \subseteq i \Rightarrow [\,]\,(\tau\,e) \neq \emptyset\} \\ \hline \left(\dfrac{[\![\mathrm{a}]\!]\,(\lambda x'.\,[\,])}{\mathsf{day}\,x'}\right)^{\downarrow} \end{array}$$

$$= \begin{array}{c} \lambda i.\,\{\_ \mid \forall x \\ \hline e.\,\mathsf{week}\,x\,e \wedge \tau\,e \subseteq i \Rightarrow [\,]\,(\tau\,e) \neq \emptyset\} \\ \hline \lambda i'.\,\bigcup\big\{[\,]\,(\tau\,e') \;\big|\; e' \in \{e' \mid \mathsf{day}\,x'\,e' \wedge \tau\,e' \subseteq i'\}\big\} \\ \hline x' \end{array}$$

$$= \begin{array}{c} \lambda i.\,\{\_ \mid \forall x \\ \hline e.\,\mathsf{week}\,x\,e \wedge \tau\,e \subseteq i \Rightarrow [\,]\,(\tau\,e) \neq \emptyset\} \\ \hline \lambda i'.\,\bigcup\big\{[\,]\,(\tau\,e') \;\big|\; \mathsf{day}\,x'\,e' \wedge \tau\,e' \subseteq i'\big\} \\ \hline x' \end{array}$$

$$[\![\text{John slept (on) one day of every week}]\!]$$

$$= ([\![\text{John slept}]\!]^{\uparrow} \backslash [\![\text{on one day of every week}]\!])^{\Downarrow}$$

$$= \left(\begin{array}{c} [\,] \\ \hline [\,] \\ \hline \mathsf{sleep\,j} \end{array} \Bigg\backslash \begin{array}{c} \lambda i.\,\{\_ \mid \forall x \\ \hline e.\,\mathsf{week}\,x\,e \wedge \tau\,e \subseteq i \Rightarrow [\,]\,(\tau\,e) \neq \emptyset\} \\ \hline \lambda i'.\,\bigcup\big\{[\,]\,(\tau\,e') \;\big|\; \mathsf{mon}\,x'\,e' \wedge \tau\,e' \subseteq i'\big\} \\ \hline \lambda a.\,a \end{array} \right)^{\Downarrow}$$

$$= \left(\begin{array}{c} \lambda i.\,\{\_ \mid \forall x \\ \hline e.\,\mathsf{week}\,x\,e \wedge \tau\,e \subseteq i \Rightarrow [\,]\,(\tau\,e) \neq \emptyset\} \\ \hline \lambda i'.\,\bigcup\big\{[\,]\,(\tau\,e') \;\big|\; \mathsf{day}\,x'\,e' \wedge \tau\,e' \subseteq i'\big\} \\ \hline \mathsf{sleep\,j} \end{array} \right)^{\Downarrow}$$

$$= \mathsf{TRUE} \ \text{ iff } \ \forall x, e.\,\mathsf{week}\,x\,e \wedge \tau\,e \subseteq i_* \Rightarrow \exists e'' \in \{e'' \mid \mathsf{sleep\,j}\,e'' \wedge \tau\,e'' \subseteq \tau\,e' \wedge \mathsf{day}\,x'\,e' \wedge \tau\,e' \subseteq \tau\,e\}$$

6 Wrapping Up

- What have we gained here?
- Clean separation of lexical content and temporal restrictions. For instance, ‘meeting’ just denotes the relation between meeting entities and meeting times. Also no events.
- Cascading restrictions are managed by the grammar. Nominals and sentence radicals are the things that get temporally restricted because those are exactly the places where the grammar needs to lower or reset.
- The scope of quantificational modifiers is now determined by familiar, robust scope-handling mechanisms, the same mechanisms that determine the scope of everything else (Barker and Shan 2014). In particular, ‘a Monday’ and ‘on a Monday’ now take scope in the same way.
- This means we can maintain a traditional bracketing for nested DPs, like [one [day of every week]].
- Conceptual connections between P&F’s interval handling operations and dynamic semantics: the intervals are a kind of dynamic context, keeping track of the “current” time under discussion. In the dG versions of the fragment, restrictors (both nominals and sentences) are a kind of anaphor (‘meeting *then*’, ‘Monday *then*’), whose reference events are determined in part by the intervals at play when they are evaluated. In the KS versions, finalizing introduces anaphoric dependencies.
- New questions: Empirically, these temporal cascades seem to be sensitive only to scope, not linear order; does factoring out the temporal restriction help avoid crossover issues? Now that the tPPs are part of the broader scope ecosystem, how should they interact with non-tPP scope-taking material, like DPs, negation, adverbs, attitudes, etc.? How the hell to add in other prepositional relations? That is, how to tie the lowering to the choice of preposition without reintroducing implicit temporal pronouns?